Power Dividers and Directional Couplers

3-port Networks (T-junctions, Power Dividers)

3-port power dividers often realized as T-junctions of transmission lines

characterized by a 3 by 3 S-matrix

limitations of 3-port networks – can a 3-port network be simultaneously reciprocal, loss-free and matched at all ports? *No*

$$\mathbf{S}=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \Rightarrow \left| \begin{array}{l} |S_{12}|^2+|S_{13}|^2=1 \\ |S_{12}|^2+|S_{23}|^2=1 \\ |S_{13}|^2+|S_{23}|^2=1 \end{array} \right. \text{ and } \left| \begin{array}{l} S_{12}^{*}S_{13}=0 \\ S_{23}^{*}S_{12}=0 \\ S_{13}^{*}S_{23}=0 \end{array} \right.$$

does not satify these equations

at least 2 of the 3 parameters S_{12}, S_{13}, S_{23} must be zero

a physically realizable 3-port device should be either unmatched on at least 1 port, or be lossy, or be non-reciprocal

3-port Networks (T-junctions, Power Dividers)

3-port power dividers can be matched on all 3 ports and can be reciprocal if they are lossy (Wilkinson divider, resistive dividers)

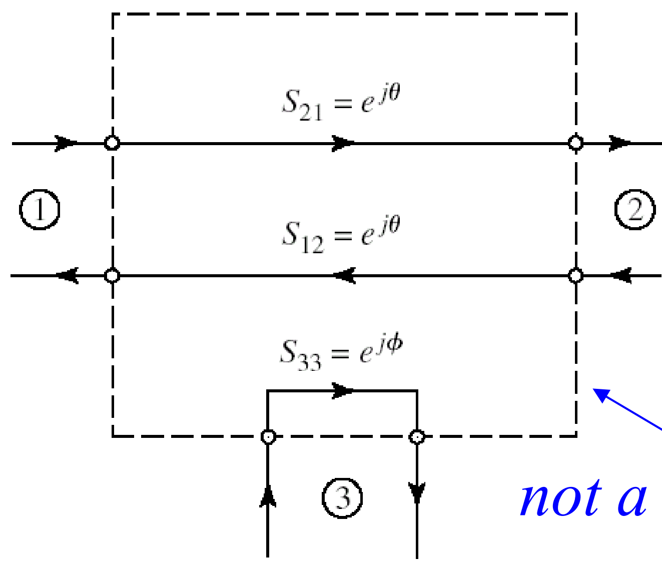
3-port devices could be loss-free and reciprocal but only 1 or 2 of the ports are matched; consider 2 matched ports

$$|S_{13}| = |S_{23}|$$

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \Rightarrow \begin{cases} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{23}|^2 = 1 \\ |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1 \end{cases} \text{ and } \begin{cases} S_{12}^* S_{13} + S_{23}^* S_{33} = 0 \\ S_{23}^* S_{12} + S_{33}^* S_{13} = 0 \\ \boxed{S_{13}^* S_{23} = 0} \end{cases}$$

$$|S_{12}| = 1 \qquad |S_{33}| = 1 \qquad |S_{13}| = |S_{23}| = 0$$

$$\Rightarrow \mathbf{S} = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$

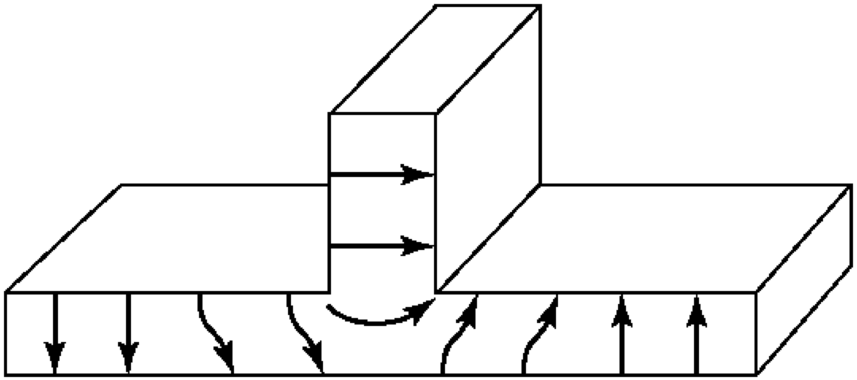


not a very useful device

T-junction Power Dividers

E-plane waveguide junction

H-plane waveguide junction



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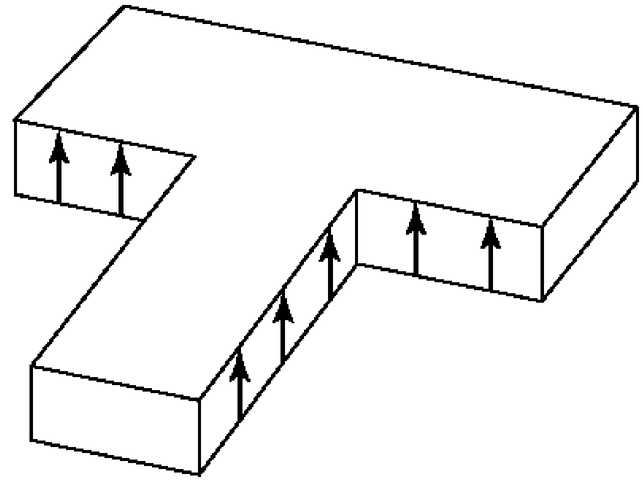

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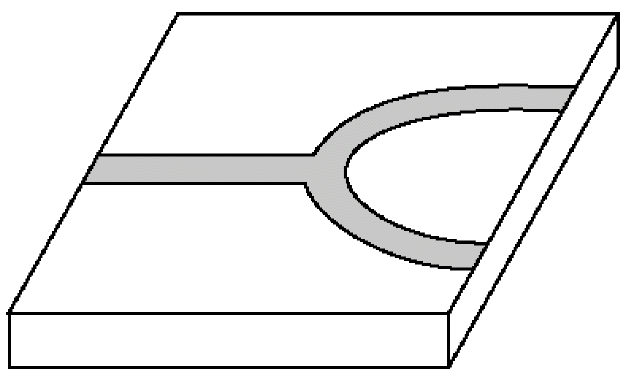


microstrip T-junction

(c)

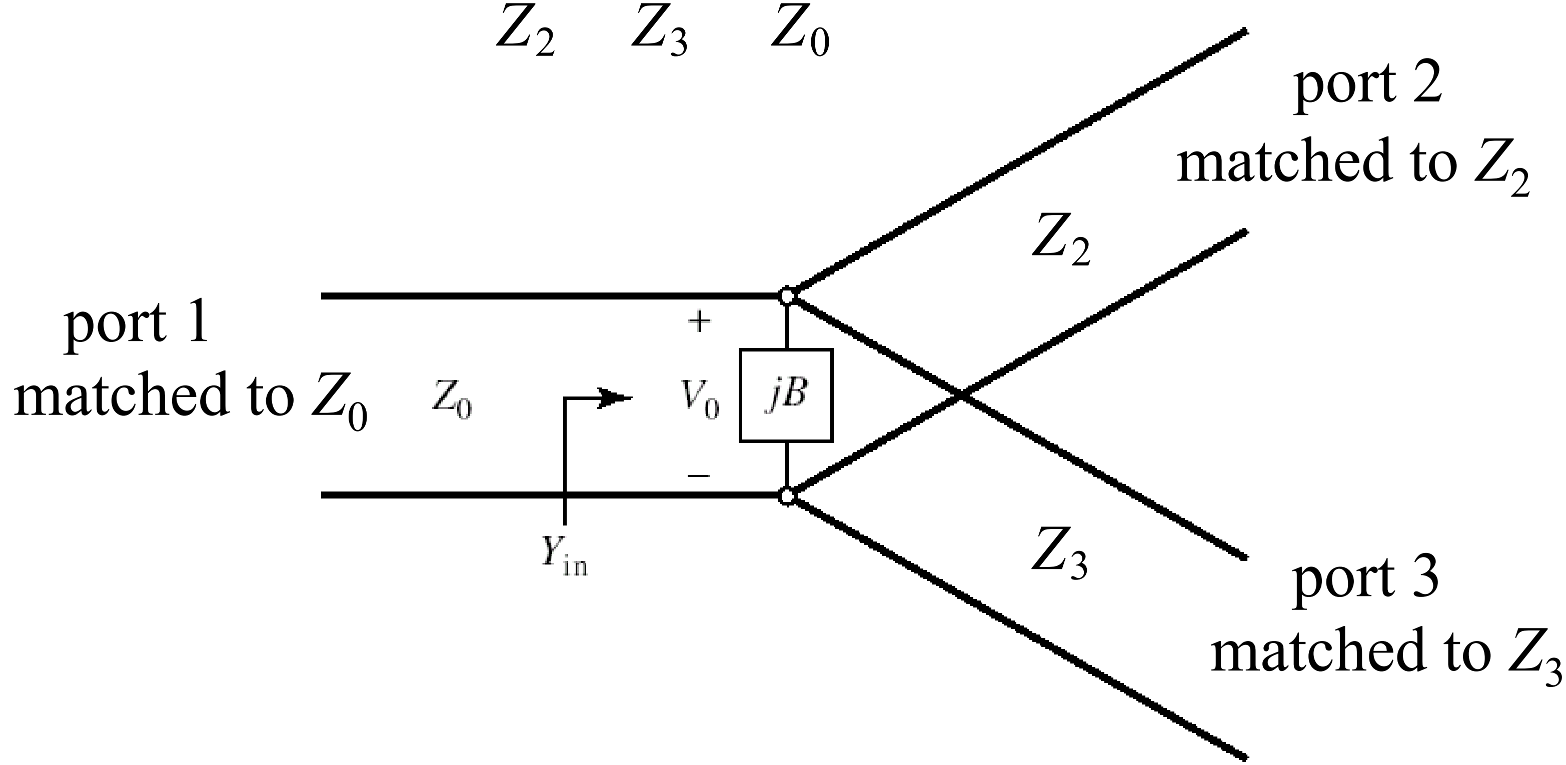
T-junction Power Dividers: TL Model

loss-free divider

$$Y_{in} = jB + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{Z_0}$$

if we ignore B (the susceptance of the junction discontinuity)

$$\frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{Z_0}$$



T-junction Power Dividers: Power Division Ratio

let $P_2 : P_3 = K : 1$

$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0},\ P_2 = \frac{1}{2}\frac{V_0^2}{Z_2},\ P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} \quad \Rightarrow \quad \boxed{\frac{P_2}{P_3} = K = \frac{Z_3}{Z_2}}$$

$$\frac{1}{Z_2} + \frac{1}{KZ_2} = \frac{1}{Z_0} \quad \Rightarrow \quad \boxed{\begin{aligned} Z_2 &= Z_0\left(1 + \frac{1}{K}\right) \\ Z_3 &= Z_0(K+1) \end{aligned}}$$

example: 3-dB power divider for $Z_0 = 50\ \Omega$

$$K = 1,\ Z_0 = 50 \Rightarrow Z_2 = Z_3 = 100$$

usually Z_2 and Z_3 are later matched to Z_0 using impedance transformers

shortcoming: the 2 output ports are not isolated from each other

T-junction Resistive Power Dividers

all ports are matched to Z_0 (advantage)

network is lossy (disadvantage)

output ports are not isolated (disadvantage)

$$Z = \frac{1}{2}\left(Z_0 + \frac{Z_0}{3}\right) = \frac{2}{3}Z_0 \Rightarrow \underline{\underline{Z_{in}}} = \frac{Z_0}{3} + Z = \frac{Z_0}{3} + \frac{2}{3}Z_0 = \underline{\underline{Z_0}}$$

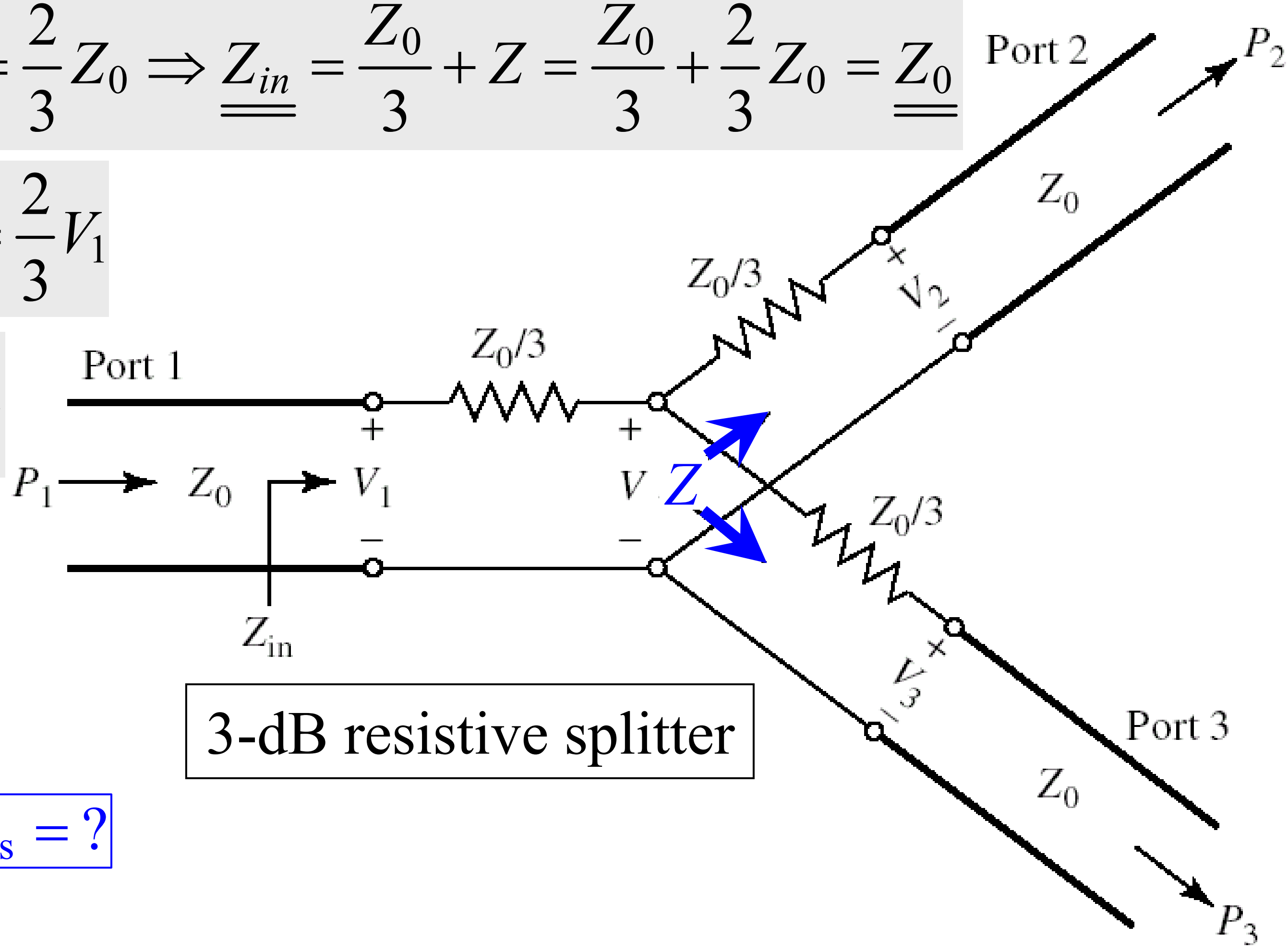
$$V = V_1 \frac{Z}{Z_0/3 + Z} = \frac{2}{3}V_1$$

$$V_2 = V_3 = \frac{3}{4}V = \frac{V_1}{2}$$

$$\Rightarrow \mathbf{S} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P_2 = P_3 = \frac{P_{in}}{4}$$

$\boxed{P_{\text{loss}} = ?}$



3-dB resistive splitter

4-port Networks (Directional Couplers)

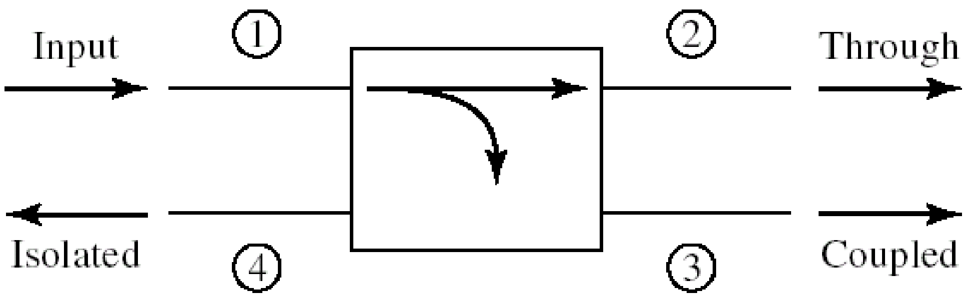
consider reciprocal matched loss-free 4-port network

$$\mathbf{S}=\begin{bmatrix} 0 & S_{12} & S_{13} & \boxed{S_{14}} \\ S_{12} & 0 & \boxed{S_{23}} & S_{24} \\ S_{13} & \boxed{S_{23}} & 0 & S_{34} \\ \boxed{S_{14}} & S_{24} & S_{34} & 0 \end{bmatrix}$$

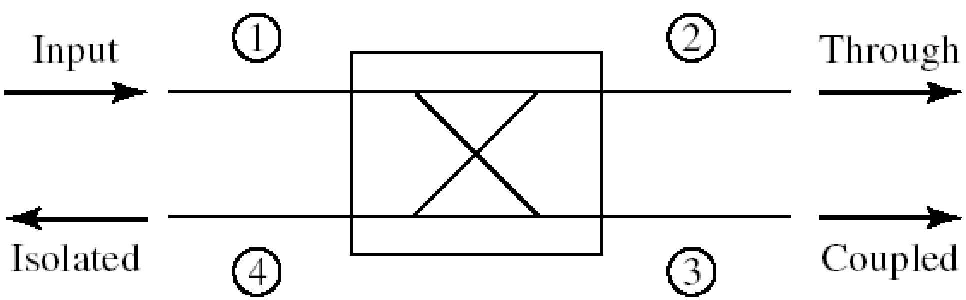
$$\left| \begin{aligned} S_{13}^*S_{23}+S_{14}^*S_{24}&=0 \\ S_{14}^*S_{13}+S_{24}^*S_{23}&=0 \end{aligned} \right. \Rightarrow S_{14}^*\left(|S_{13}|^2-|S_{24}|^2\right)=0$$

$$\left| \begin{aligned} S_{12}^*S_{23}+S_{14}^*S_{34}&=0 \\ S_{14}^*S_{12}+S_{34}^*S_{23}&=0 \end{aligned} \right. \Rightarrow S_{23}^*\left(|S_{12}|^2-|S_{34}|^2\right)=0$$

directional-coupler solution $S_{14}=S_{23}=0$



$$\left| \begin{aligned} |S_{12}|^2+|S_{13}|^2&=1 \\ |S_{12}|^2+|S_{24}|^2&=1 \\ |S_{13}|^2+|S_{34}|^2&=1 \\ |S_{24}|^2+|S_{34}|^2&=1 \end{aligned} \right.$$



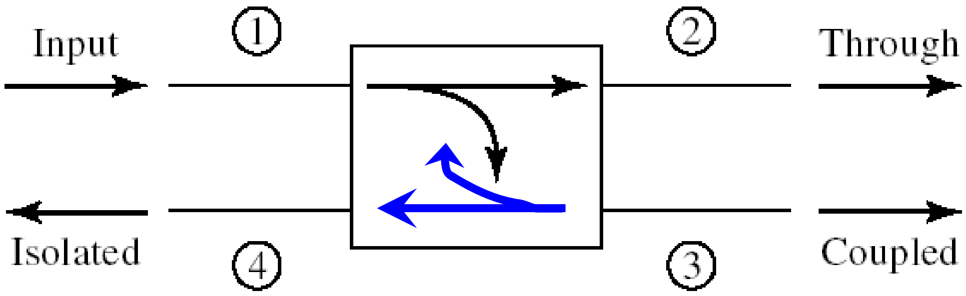
$$\Rightarrow \left| \begin{aligned} |S_{13}|&=|S_{24}| \\ |S_{12}|&=|S_{34}| \end{aligned} \right.$$

Directional Couplers: Scattering Matrix

choose reference planes so that

$$S_{12} = S_{34} = \alpha$$
$$S_{13} = \beta e^{jB_1},\ S_{24} = \beta e^{jB_2}$$

$$\alpha^2 + \beta^2 = 1$$



$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \ \Rightarrow\ B_1 + B_2 = \pi \pm 2n\pi \qquad (\text{set } n = 0)$$

case 1: $B_1 = B_2 = \pi/2$

$$\mathbf{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

symmetrical coupler

case 2: $B_1 = 0,\ B_2 = \pi$

$$\mathbf{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

anti-symmetrical coupler

Directional Couplers: Scattering Matrix (2)

similar result is obtained if we choose the reference planes so that

$$S_{12} = \alpha e^{jA_1},\ S_{34} = \alpha e^{jA_2}$$
$$S_{13} = S_{24} = \beta$$

$$\alpha^2 + \beta^2 = 1$$

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \ \Rightarrow \ A_1 + A_2 = \pi \pm 2n\pi \qquad (\text{set } n = 0)$$

case 1: $A_1 = A_2 = \pi/2$

$$\mathbf{S} = \begin{bmatrix} 0 & j\alpha & \beta & 0 \\ j\alpha & 0 & 0 & \beta \\ \beta & 0 & 0 & j\alpha \\ 0 & \beta & j\alpha & 0 \end{bmatrix}$$

case 2: $A_1 = 0,\ A_2 = \pi$

$$\mathbf{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & \beta \\ \beta & 0 & 0 & -\alpha \\ 0 & \beta & -\alpha & 0 \end{bmatrix}$$

always: if the "through" parameters are in phase, the phases of the "couple" parameters must add to π and *vice versa*

Directional Couplers: Scattering Matrix (3)

$$\begin{cases} S_{14}^*\left(|S_{13}|^2 - |S_{24}|^2\right) = 0 \\ S_{23}^*\left(|S_{12}|^2 - |S_{34}|^2\right) = 0 \end{cases}$$

another solution to the above equations: $|S_{13}| = |S_{24}|, \quad |S_{12}| = |S_{34}|$

this is also satisfied by the directional-coupler solution
but here we also assume that $|S_{14}| \neq 0, \quad |S_{23}| \neq 0$ (no isolation)

from the unitary conditions

$$\begin{cases} |S_{12}|^2 + \boxed{|S_{13}|^2} + |S_{14}|^2 = 1 \\ |S_{12}|^2 + \boxed{|S_{24}|^2} + |S_{23}|^2 = 1 \\ \boxed{|S_{13}|^2} + |S_{34}|^2 + |S_{23}|^2 = 1 \\ \boxed{|S_{24}|^2} + |S_{34}|^2 + |S_{14}|^2 = 1 \end{cases} \quad \Rightarrow \quad |S_{14}| = |S_{23}| \neq 0$$

Directional Couplers: Scattering Matrix (4)

choose plane references such that "through" S-parameters are in phase

$$S_{12} = S_{34} = \alpha$$
$$S_{13} = \beta e^{jB_1},\ S_{24} = \beta e^{jB_2}$$

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \ \Rightarrow\ B_1 + B_2 = \pi \pm 2n\pi \qquad (\text{set } n = 0)$$

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \ \Rightarrow\ e^{-jB_1} S_{23} + e^{j(\pi - B_1)} S_{14}^* = 0 \ \Rightarrow\ \underline{S_{23} - S_{14}^* = 0}$$
$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \ \Rightarrow\ \underline{S_{23} + S_{14}^* = 0} \qquad \Rightarrow \qquad S_{14} = S_{23} = 0$$

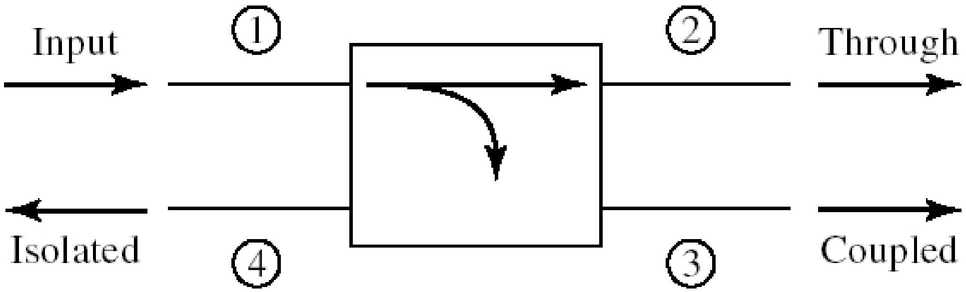
the same result would be obtained if we set reference planes so that

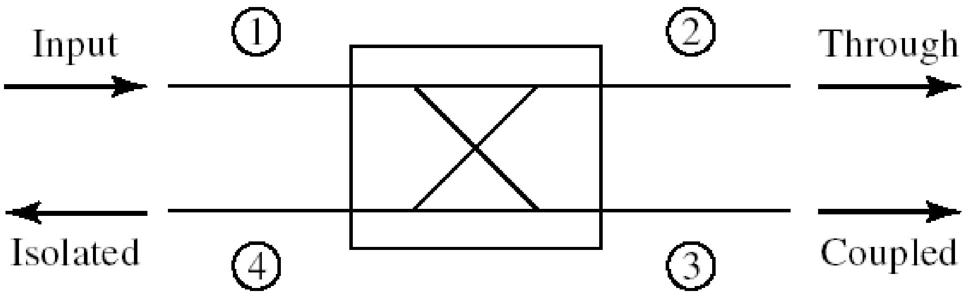
$$S_{12} = \alpha e^{jA_1},\ S_{34} = \alpha e^{jA_2}$$
$$S_{13} = S_{24} = \beta$$

the directional-coupler solution is after all the only solution

a reciprocal, loss-free, matched 4-port network is always a directional coupler with one pair of ports decoupled (input/isolated)

Directional Couplers: Performance Parameters





Coupling: $C = 10\log_{10}\dfrac{P_1}{P_3} = -20\log_{10}\underbrace{|S_{31}|}_{\beta}$ dB

Directivity: $D = 10\log_{10}\dfrac{P_3}{P_4} = 20\log_{10}\dfrac{|S_{31}|}{|S_{41}|}$ dB

Isolation: $I = 10\log_{10}\dfrac{P_1}{P_4} = -20\log_{10}|S_{41}|$ dB

$I = D + C$, dB

<u>the ideal coupler</u>

$I \to \infty,\ D \to \infty$

Hybrid Couplers

a particular case of a directional coupler with C = 3 dB (equal power split between the *through* and *coupled* ports)

$$\alpha = \beta = 1/\sqrt{2}$$

S-matrix of the ***quadrature hybrid*** (symmetrical coupler of C = 3 dB): has 90° phase shift between the *through* and *coupled* ports

$$\mathbf{S} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

anti-symmetrical hybrid: has 0° phase shift between the *through* and *coupled* ports if ports 1 or 3 are excited (but has 180° phase shift if ports 2 or 4 are excited)

$$\mathbf{S} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

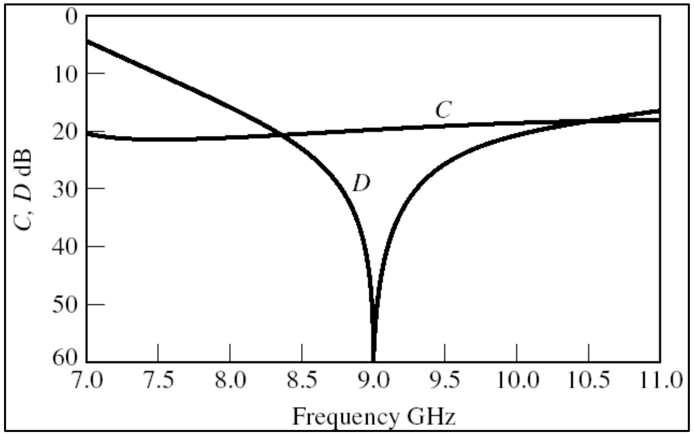
Bethe Hole Coupler

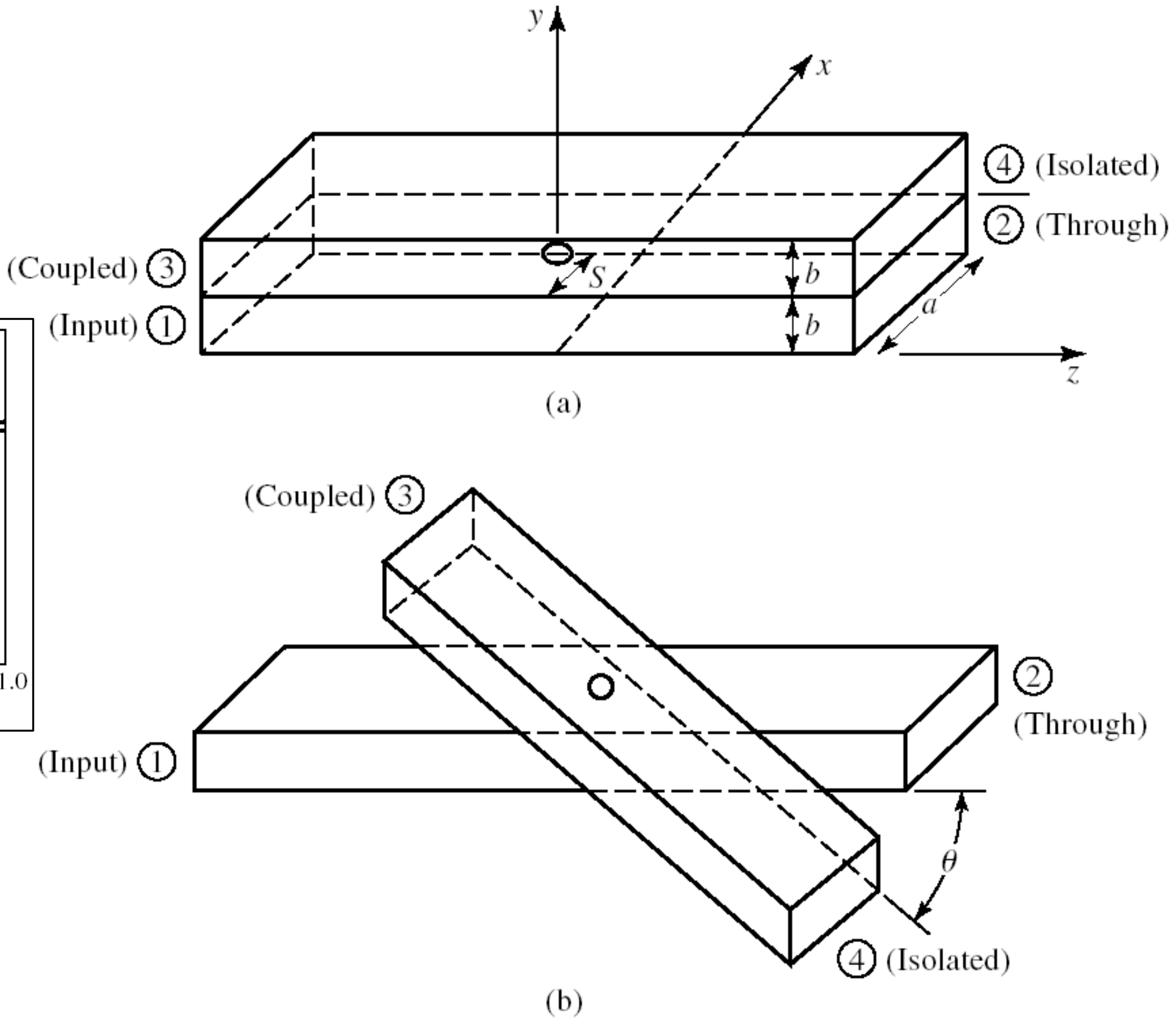
simple design (advantage)

narrow-band performance (disadvantage)

coupling is through a
hole in the broad wall

weak coupling





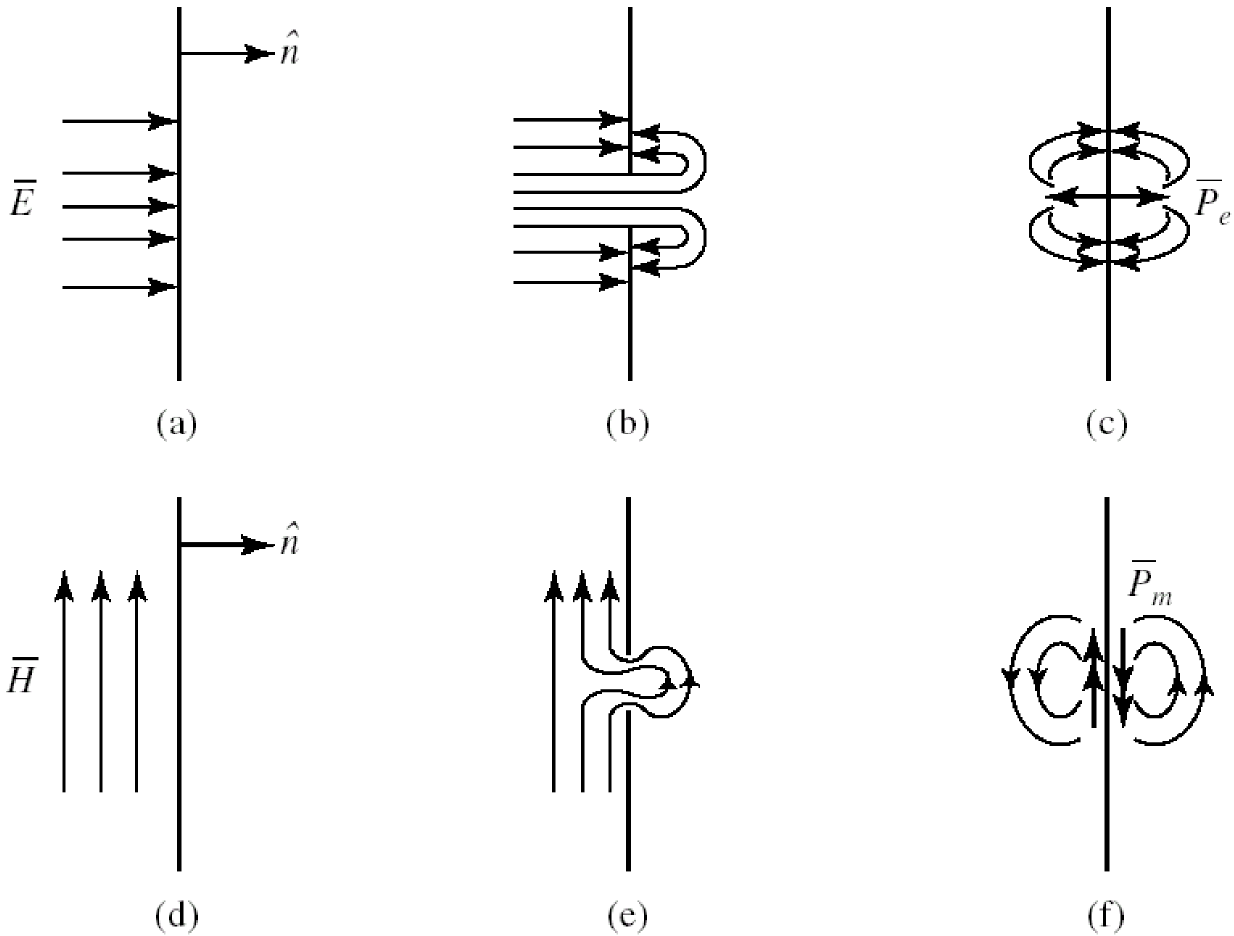
Bethe Hole Coupler: Excitation through a Hole

$$\mathbf{J} = j\omega\mathbf{P}_e = \hat{\mathbf{n}} j\omega\varepsilon_0\alpha_e E_n \delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

$$\mathbf{M} = j\omega\mu_0\mathbf{P}_m = -j\omega\mu_0\alpha_m \mathbf{H}_{\tan}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

$$\mathbf{P}_e = \varepsilon_0\chi_e\mathbf{E}$$

$$\mathbf{P}_m = \chi_m\mathbf{H}$$



polarizabilities α_e and α_m depend on the size and shape of the hole

circular hole:

$$\alpha_e = \frac{2r_0^3}{3}, \alpha_m = \frac{4r_0^3}{3}$$

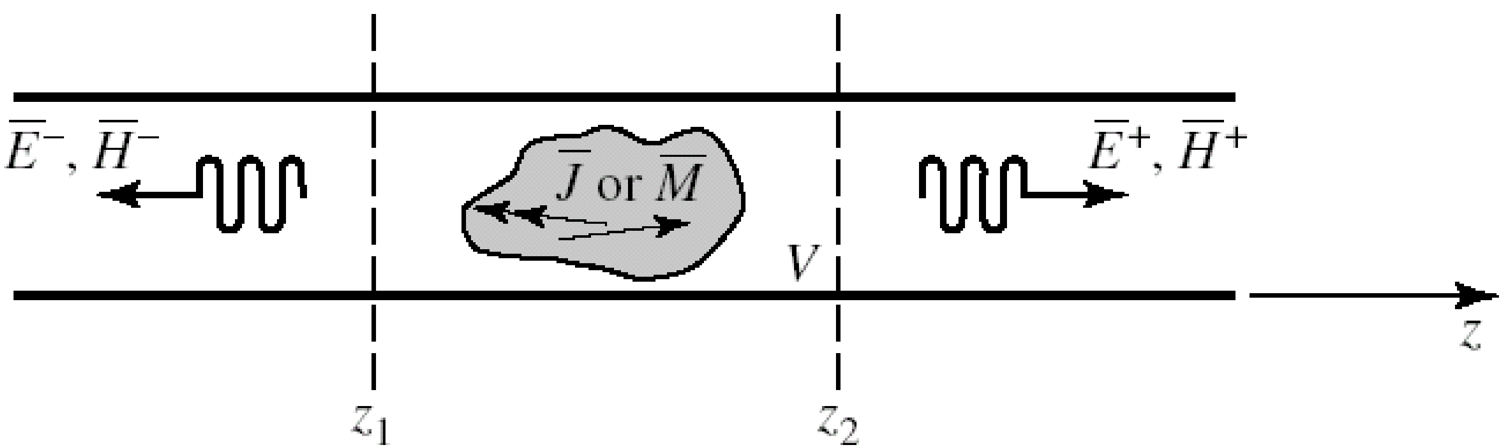
Bethe Hole Coupler: Excitation through a Hole (2)

using the Lorentz reciprocity theorem, we find the amplitudes of the excited forward and backward modal waves

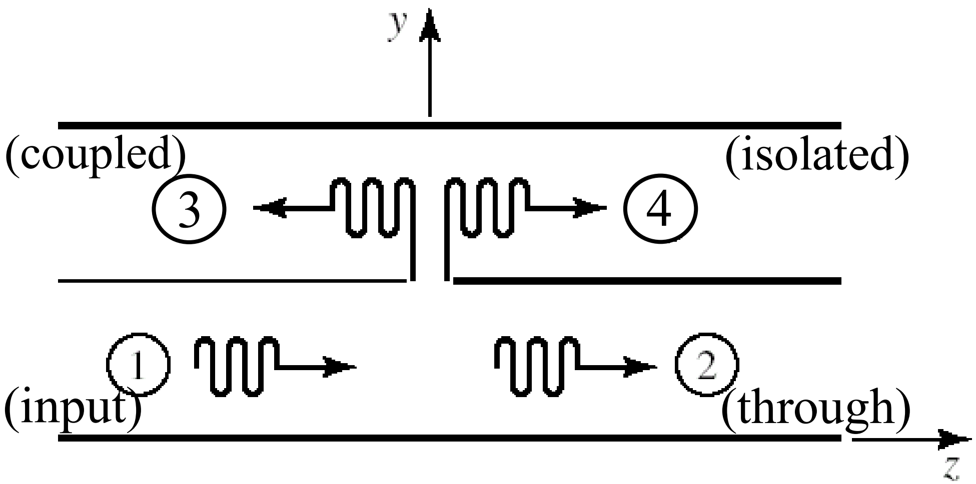
$$A_n^+ = \frac{1}{P_n}\iiint_V \left[(-\mathbf{h}_n + \hat{\mathbf{z}}h_{zn})\cdot\mathbf{M} - (\mathbf{e}_n - \hat{\mathbf{z}}e_{nz})\cdot\mathbf{J}\right]e^{j\beta_n z}dv$$

$$A_n^- = \frac{1}{P_n}\iiint_V \left[(\mathbf{h}_n + \hat{\mathbf{z}}h_{zn})\cdot\mathbf{M} - (\mathbf{e}_n + \hat{\mathbf{z}}e_{nz})\cdot\mathbf{J}\right]e^{-j\beta_n z}dv$$

$$P_n = 2\underbrace{\iint_S (\mathbf{e}_n \times \mathbf{h}_n)\cdot\hat{\mathbf{z}}ds}_{1}$$



Bethe Hole Coupler: Excitation through a Broad-side Hole



incident field is known, e.g., for the dominant TE_{10} mode

$$E_y = A\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_x = -\frac{A}{Z_{10}}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

- find the field at the center of the hole
- find the respective currents
- find the excited modes in the upper guide (integrals very simple due to the δ-function sources)
- set $A^+ = 0$ to find position of hole s

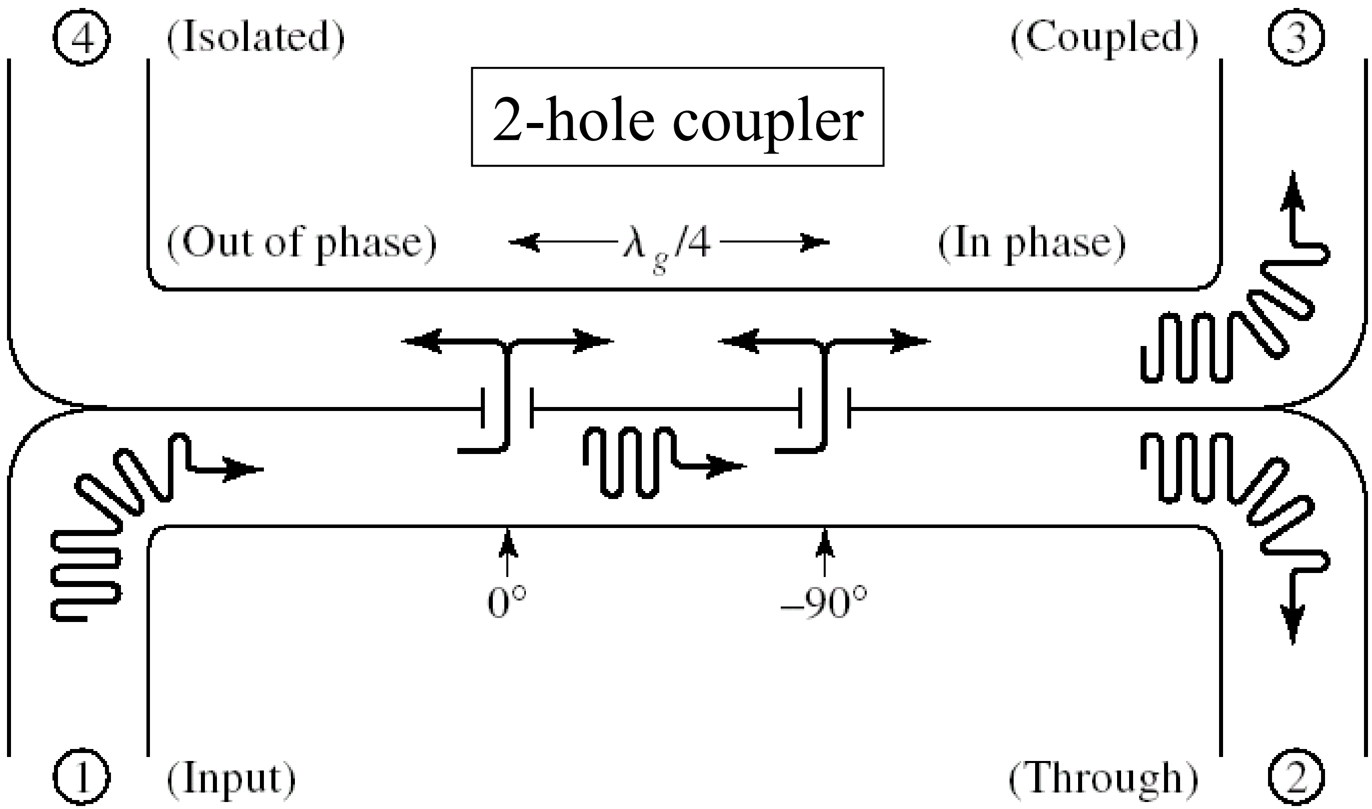
for TE_{10} mode:

$$\sin\left(\frac{\pi s}{a}\right) = \frac{\lambda_0}{\sqrt{2(\lambda_0^2 - a^2)}}$$

Multi-hole Couplers

coupling is in the forward direction

broadband performance achieved with multi-hole couplers



Multi-hole Couplers (2)

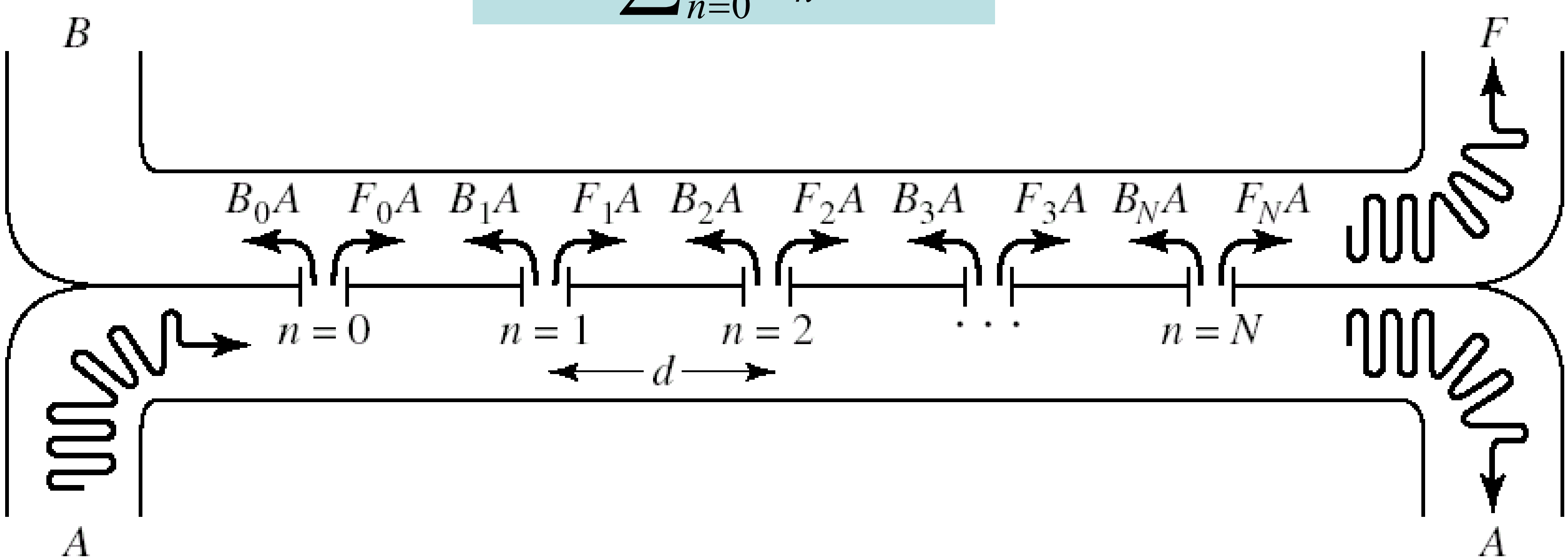
A is the amplitude of the incident wave in the lower waveguide and *approximately* the amplitude of the wave at the "through" port

F_n is the forward coupling coefficient of the nth aperture, ~ α_e and α_m

B_n is the backward coupling coefficient of the nth aperture, ~ α_e and α_m

$$F = Ae^{-j\beta Nd}\sum_{n=0}^{N} F_n$$

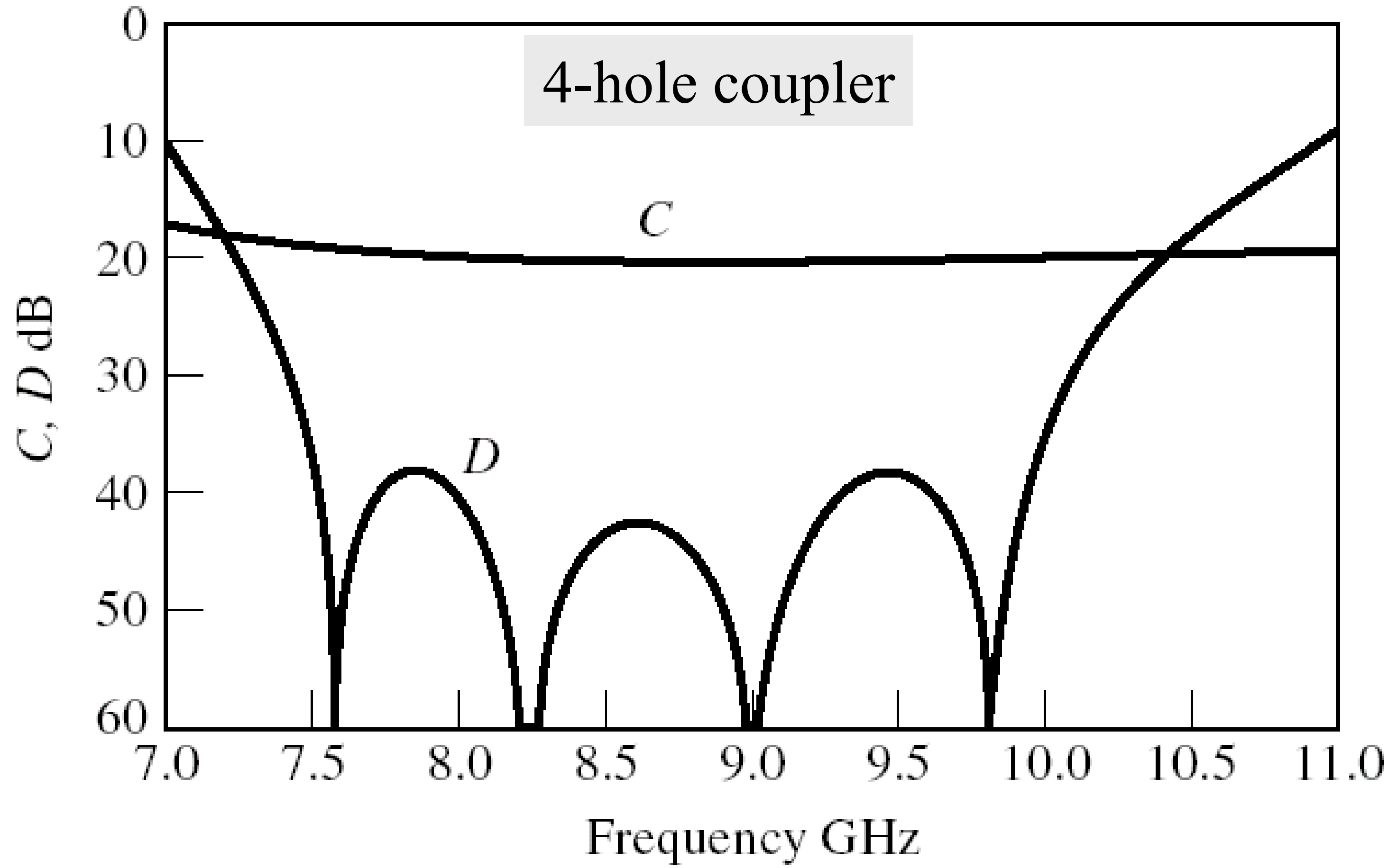
$$B = A\sum_{n=0}^{N} B_n e^{-j2\beta nd}$$



Multi-hole Couplers (3)

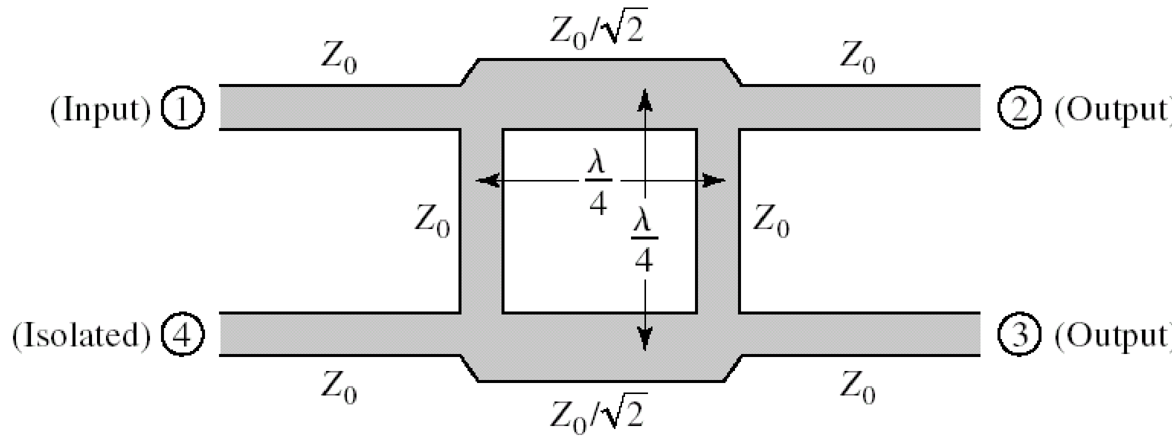
couplers of binomial and Chebyshev coupling can be designed

usually *s* is kept the same for all holes while their radii are properly chosen

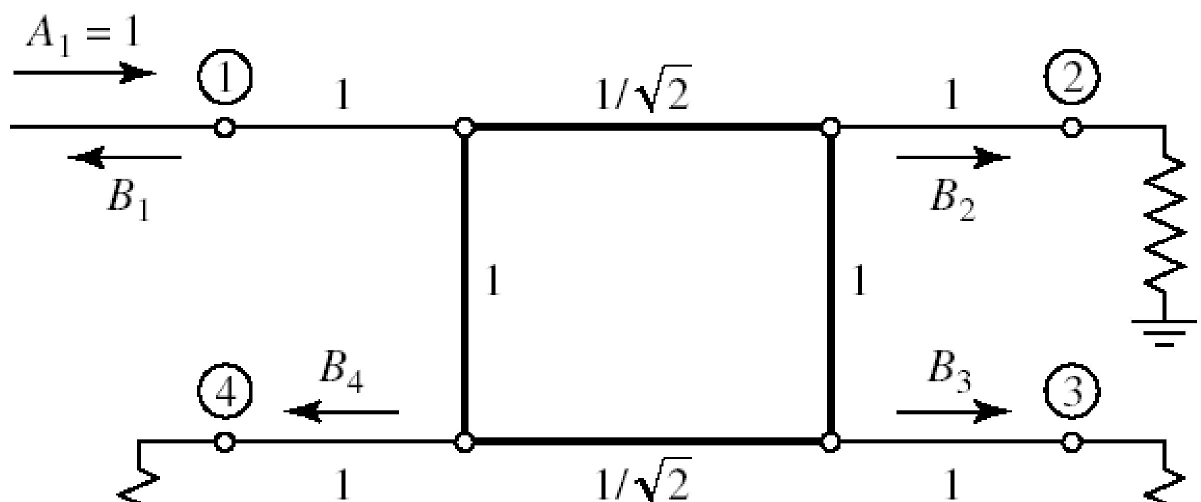


Quadrature Hybrid

$$\mathbf{S} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

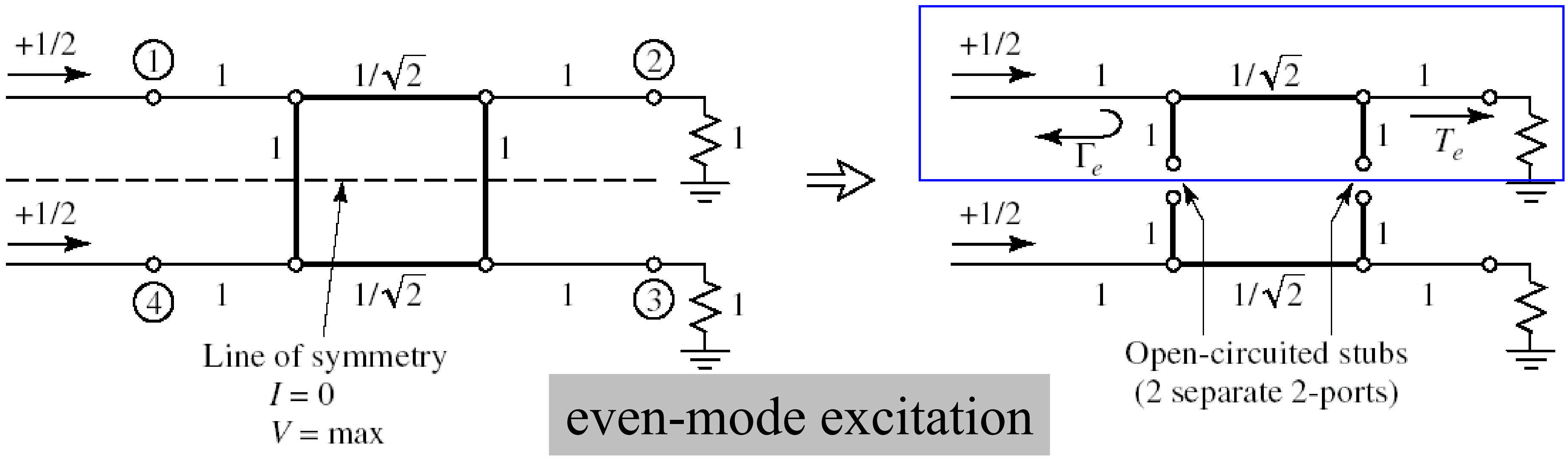


even-odd mode analysis of the equivalent circuit helps understand how the hybrid works



equivalent circuit (normalized to Z_0)

Quadrature Hybrid: Even-Odd Mode Analysis



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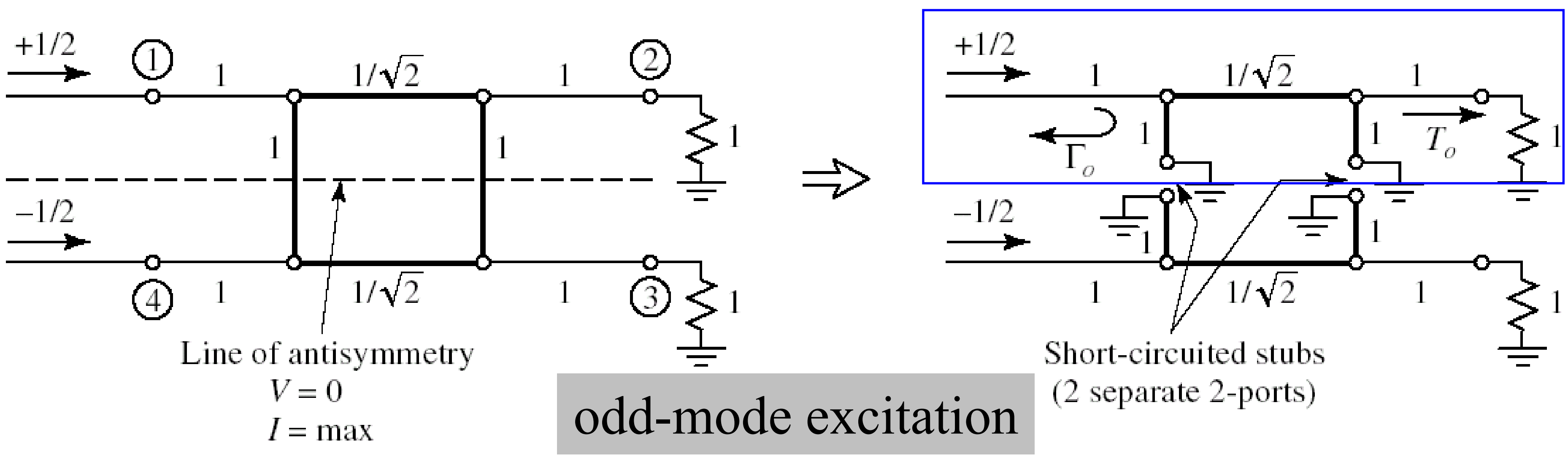
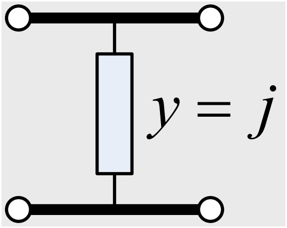


(b)

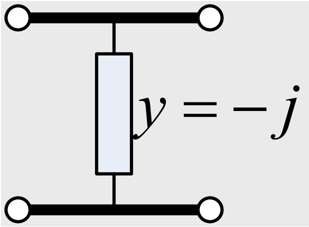
Quadrature Hybrid: Even-Odd Mode Analysis (2)

$$B_1 = 0.5(\Gamma_e + \Gamma_o),\ B_2 = 0.5(T_e + T_o),\ B_3 = 0.5(T_e - T_o),\ B_4 = 0.5(\Gamma_e - \Gamma_o)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \underbrace{\begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{OC stub} \\ l=\lambda/8}} \underbrace{\begin{bmatrix} 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}}_{\lambda/4\ \text{TL}} \underbrace{\begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{OC stub} \\ l=\lambda/8}} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \underbrace{\begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{SC stub} \\ l=\lambda/8}} \underbrace{\begin{bmatrix} 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}}_{\lambda/4\ \text{TL}} \underbrace{\begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}}_{\substack{\text{shunt} \\ \text{SC stub} \\ l=\lambda/8}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$



$$\Gamma = S_{11} = \frac{A+B-C-D}{A+B+C+D} = 0$$

$$T = S_{21} = \frac{2}{A+B+C+D} = -\frac{1+j}{\sqrt{2}}$$

$\Rightarrow$

$$\Gamma_e = 0,\ T_e = -\frac{(1+j)}{\sqrt{2}}$$

$$\Gamma_o = 0,\ T_o = \frac{1-j}{\sqrt{2}}$$

$\Rightarrow$

$$B_1 = 0$$

$$B_2 = -j/\sqrt{2}$$

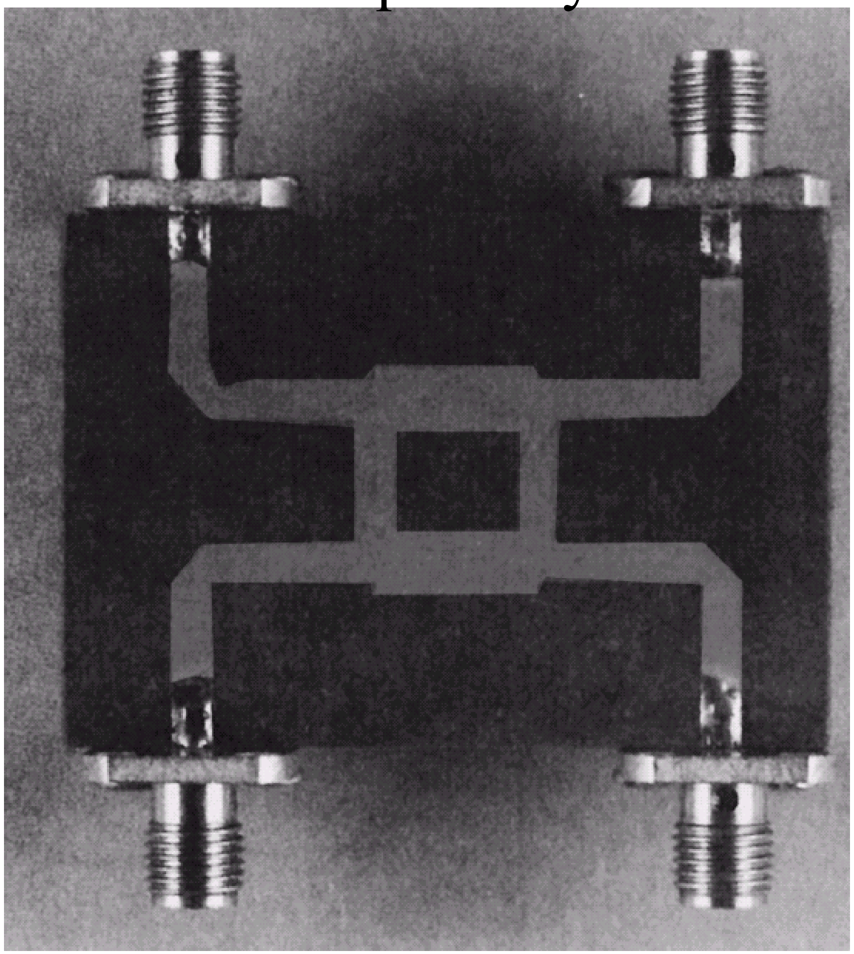
$$B_3 = -1/\sqrt{2}$$

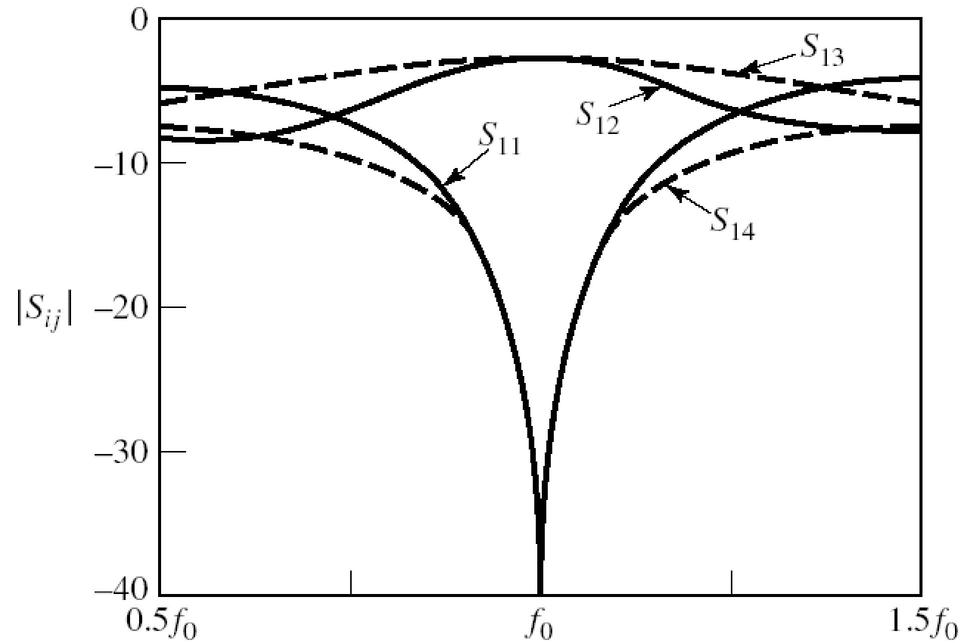
$$B_4 = 0$$

Quadrature Hybrid Performance

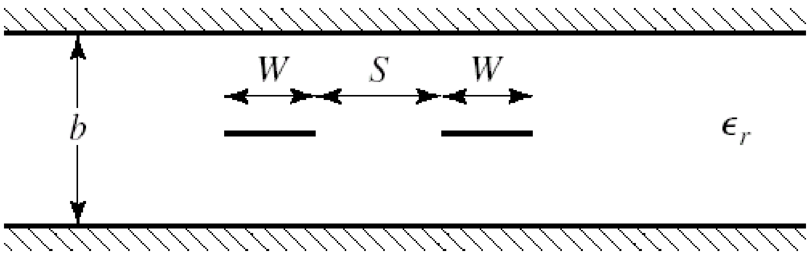
relatively narrow-band (10 to 20 %)

microstrip 90° hybrid



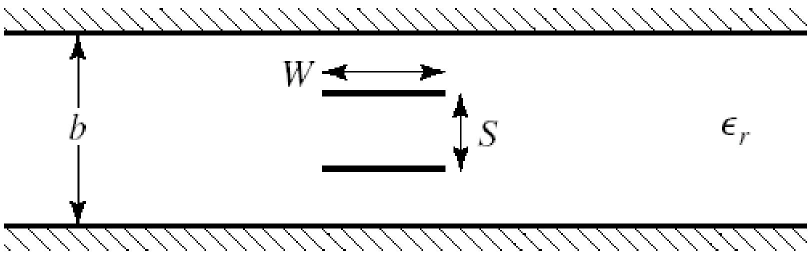


Coupled-line Directional Couplers: Coupled Lines



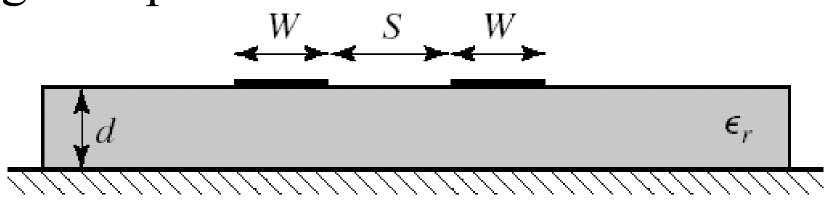
(a)

strip lines / edge-coupled



(b)

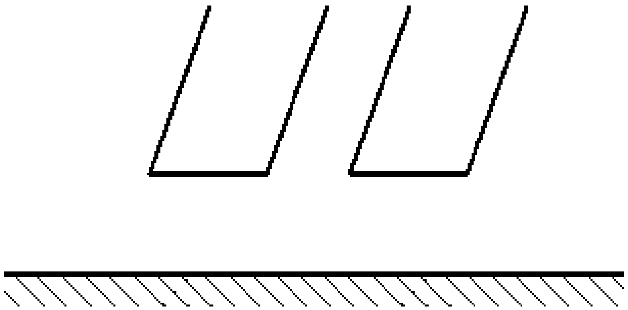
strip lines / broadside-coupled

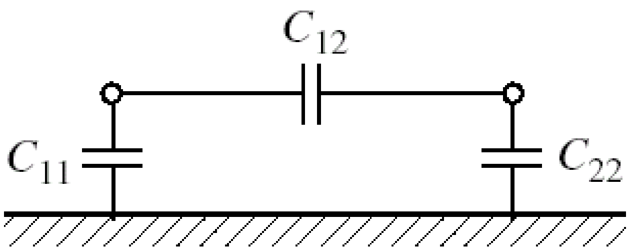


(c)

microstrip lines / edge-coupled

for TEM propagation

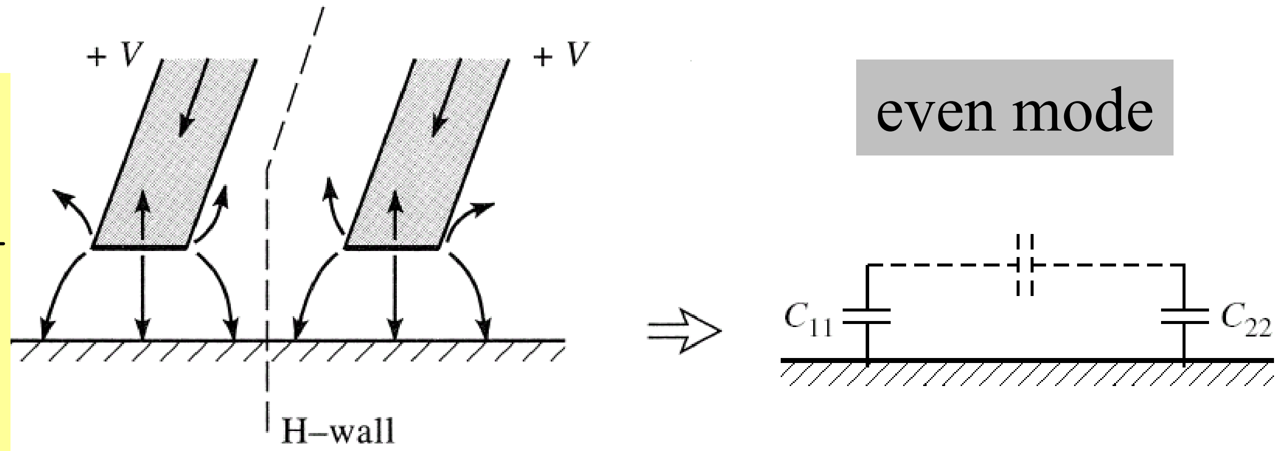




Coupled Lines: Even / Odd Mode Analysis (Symmetric Lines)

$$C_e = C_{11} = C_{22}$$

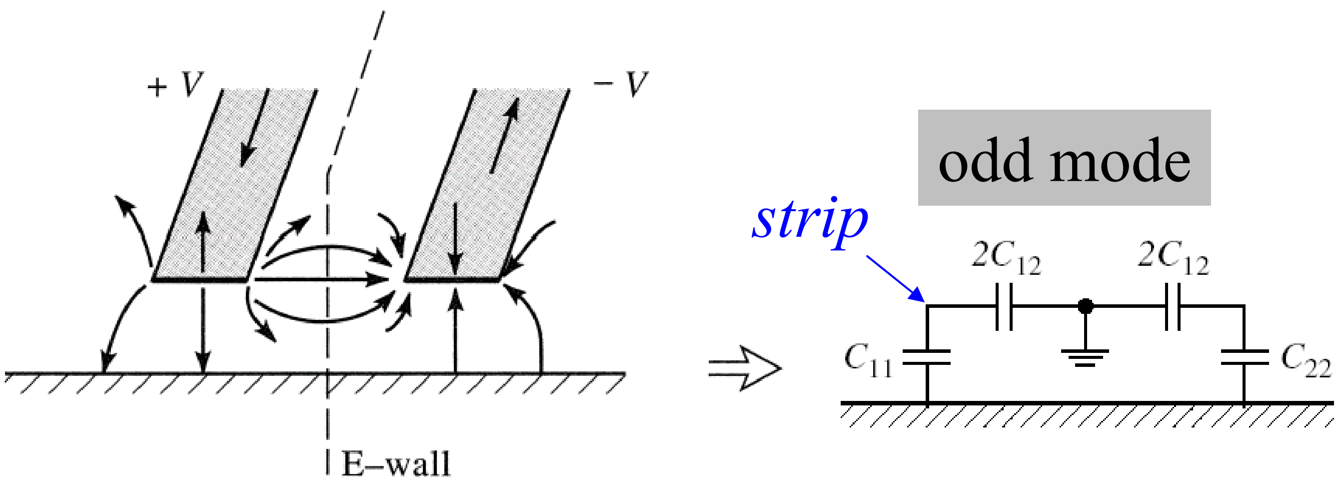
$$Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{v_p C_e}$$



(a)

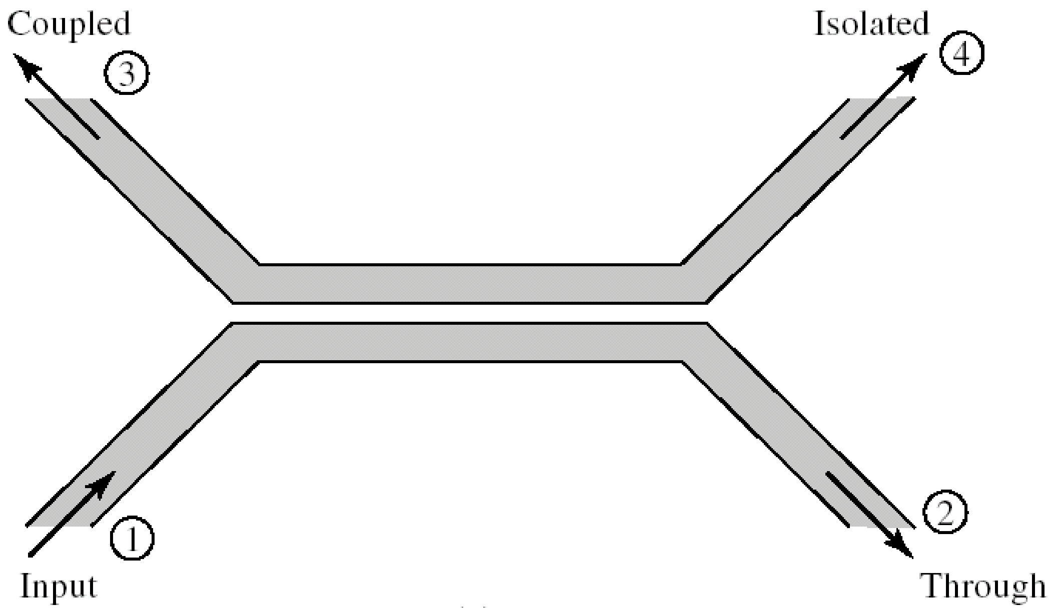
$$C_o = C_{11} + 2C_{12}$$

$$Z_{0o} = \frac{1}{v_p C_o}$$

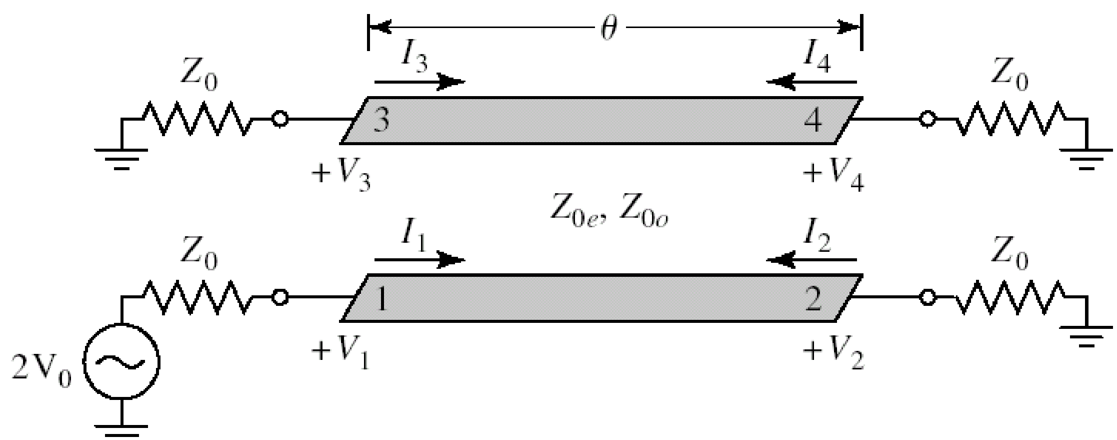


(b)

Single-section Coupled-line Directional Coupler



(a)



(b)

Single-section Coupled-line Coupler: Even / Odd Mode Analysis

$$I_1^e = I_3^e, I_2^e = I_4^e$$
$$V_1^e = V_3^e, V_2^e = V_4^e$$

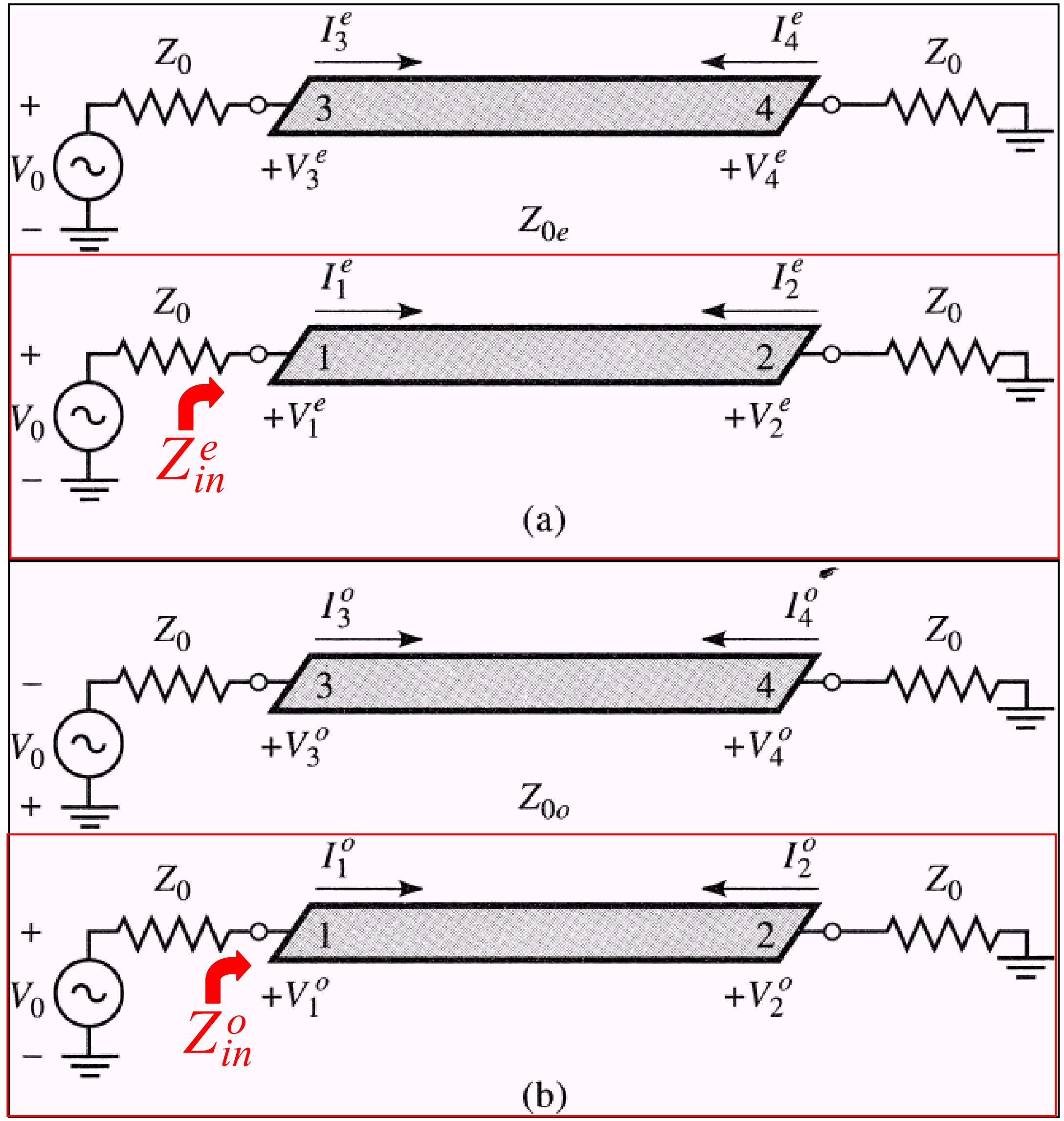
$$Z_{in}^e = Z_{0e}\frac{Z_0 + jZ_{0e}\tan(\beta L)}{Z_{0e} + jZ_0\tan(\beta L)}$$

$$V_1^e = V_0\frac{Z_{in}^e}{Z_{in}^e + Z_0}, I_1^e = \frac{V_0}{Z_{in}^e + Z_0}$$

$$I_1^e = -I_3^e, I_2^e = -I_4^e$$
$$V_1^e = -V_3^e, V_2^e = -V_4^e$$

$$Z_{in}^o = Z_{0o}\frac{Z_0 + jZ_{0o}\tan(\beta L)}{Z_{0o} + jZ_0\tan(\beta L)}$$

$$V_1^o = V_0\frac{Z_{in}^o}{Z_{in}^o + Z_0}, I_1^o = \frac{V_0}{Z_{in}^o + Z_0}$$



Coupled-line Coupler: Even / Odd Mode Analysis (2)

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} \Rightarrow \underline{Z_{in} = Z_0 + \frac{2(Z_{in}^e Z_{in}^o - Z_0^2)}{Z_{in}^e + Z_{in}^o + 2Z_0}}$$

set the characteristic impedance of the (single) TL

$$Z_0 = \sqrt{Z_{0e} Z_{0o}} \Rightarrow Z_{in} = Z_0$$

the *coupled*-port voltage is then

$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = V_0 \left(\frac{Z_{in}^e}{Z_{in}^e + Z_0} - \frac{Z_{in}^o}{Z_{in}^o + Z_0} \right)$$

$$\Rightarrow \underline{\underline{V_3 = V_0 j \frac{C \tan(\beta L)}{\sqrt{1 - C^2} + j \tan(\beta L)}}}, \text{ where } C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

Coupled-line Coupler: Even / Odd Mode Analysis (3)

the *through*-port voltage is obtain in a similar way

$$\underline{\underline{V_2 = V_0 \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos(\beta L) + j\sin(\beta L)}}}$$

the isolation-port voltage is obtained as $V_4 = 0$

if $L = \lambda/4$, then

$$\frac{V_3}{V_0} = C,\ \frac{V_2}{V_0} = -j\sqrt{1-C^2},\ V_4 = 0 \qquad C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

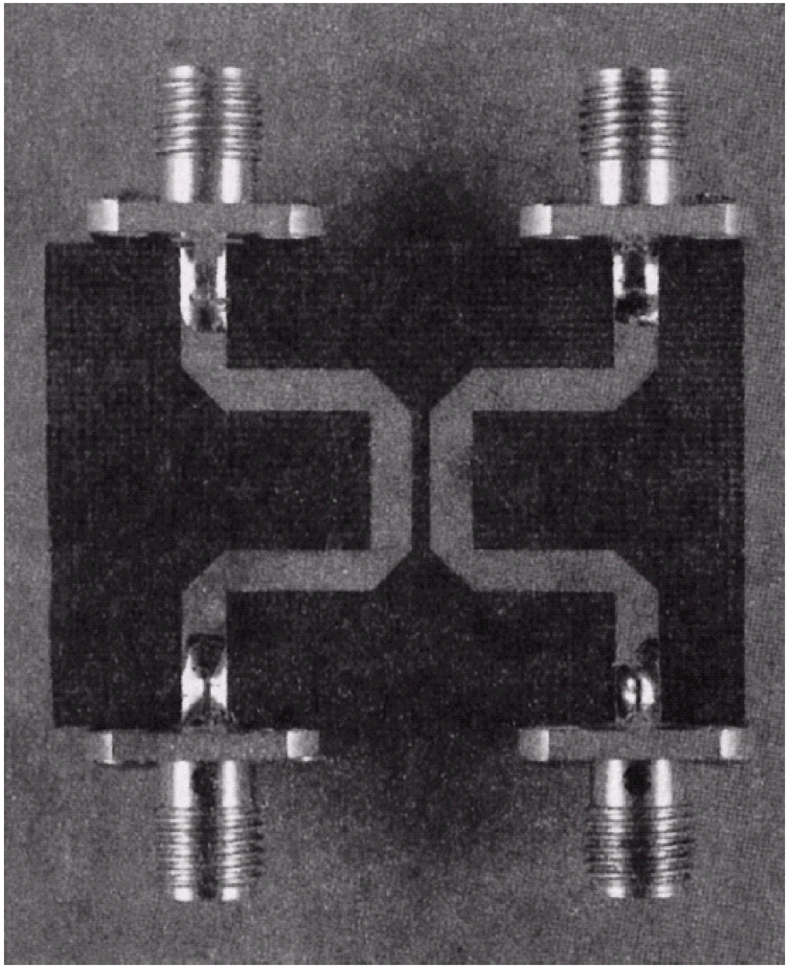
coupling

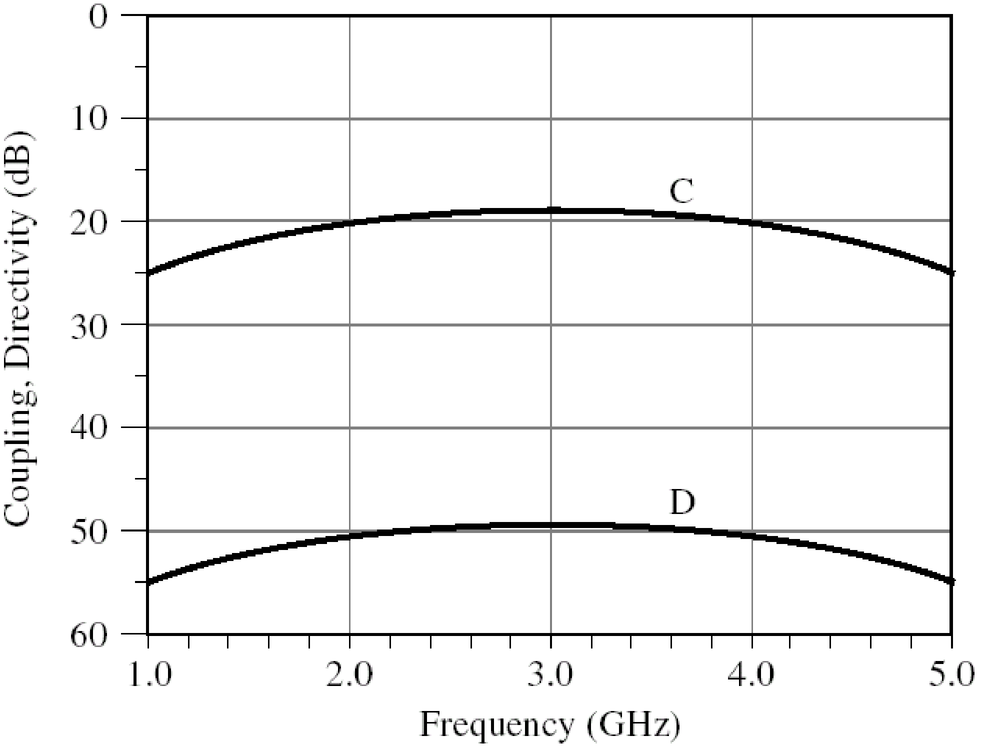
for given Z_0 and C

$$\left| \begin{array}{l} Z_0 = \sqrt{Z_{0e} Z_{0o}} \\ C = \dfrac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \end{array} \right. \Rightarrow \left| \begin{array}{l} Z_{0e} = Z_0 \sqrt{\dfrac{1+C}{1-C}} \\ Z_{0o} = Z_0 \sqrt{\dfrac{1-C}{1+C}} \end{array} \right.$$

Single-section Coupled-line Coupler

microstrip single-section coupler



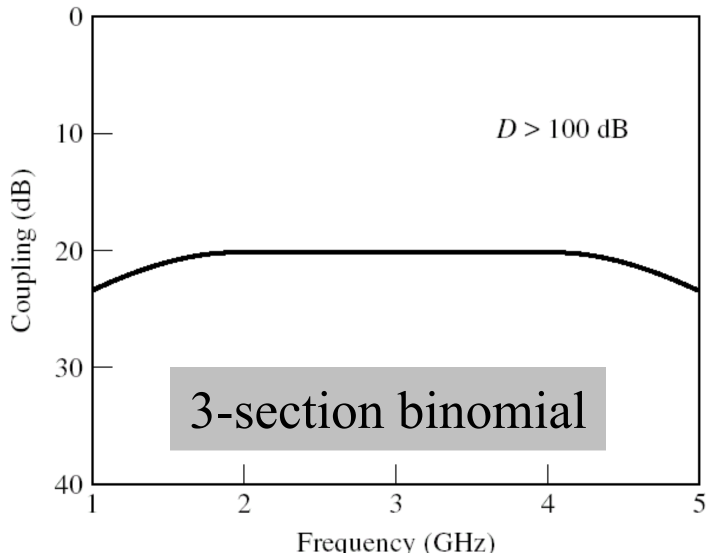


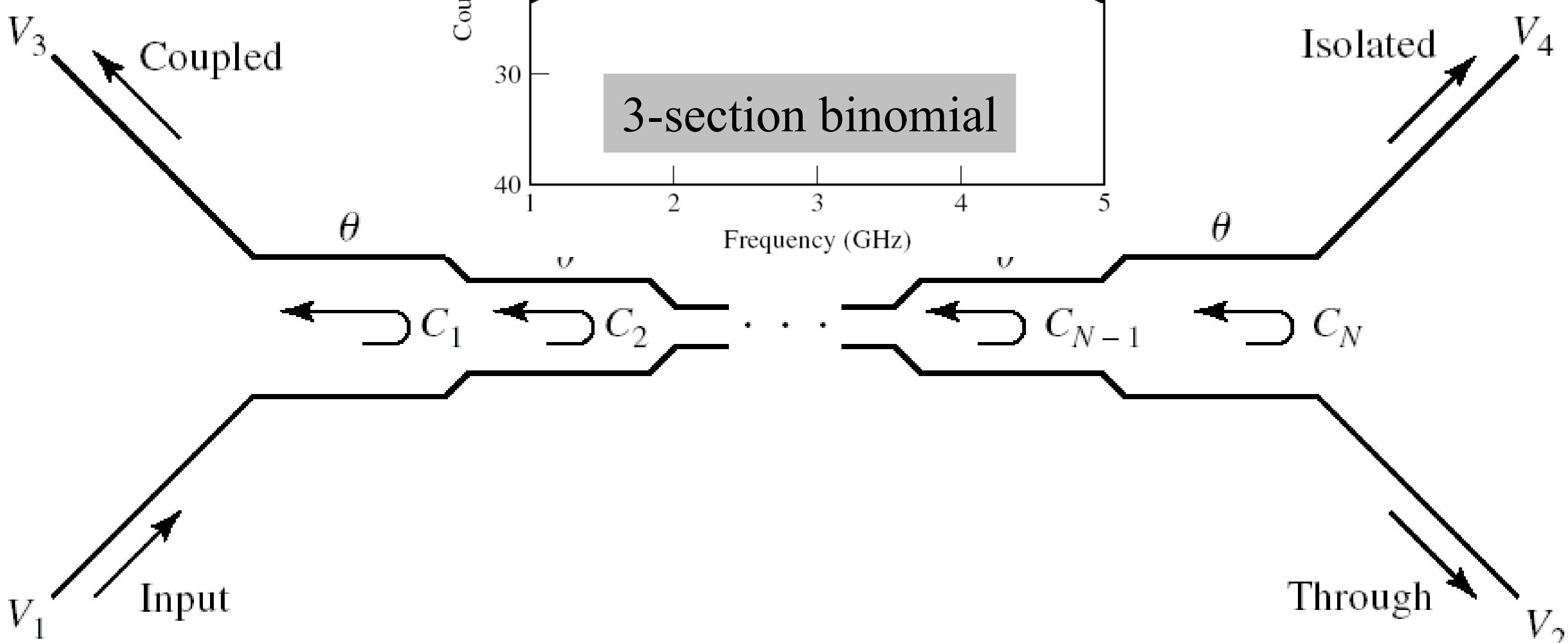
Multi-section Coupled-line Couplers

broadband performance (decade bandwidths)

low coupling levels (same as with multi-hole waveguide couplers)

same phase velocity for even and odd modes is important





Lange Couplers

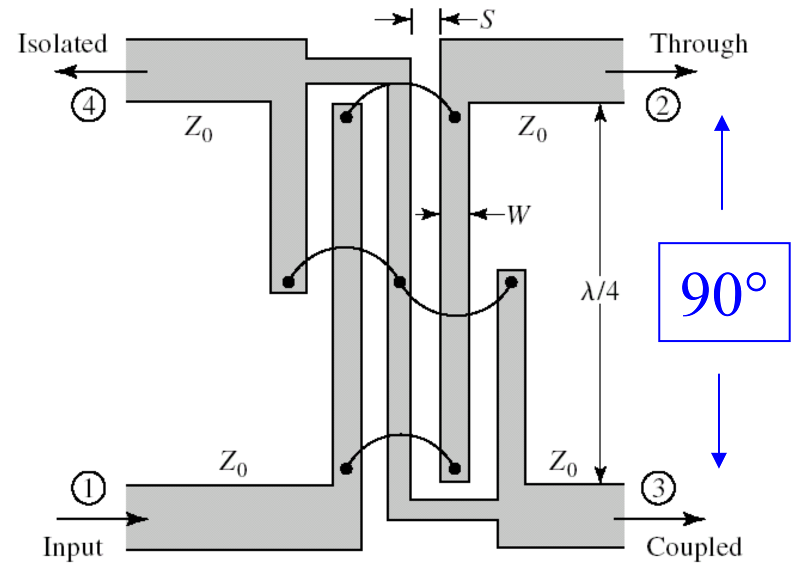
interdigitated line geometry

provide strong coupling: 6 dB, 3dB

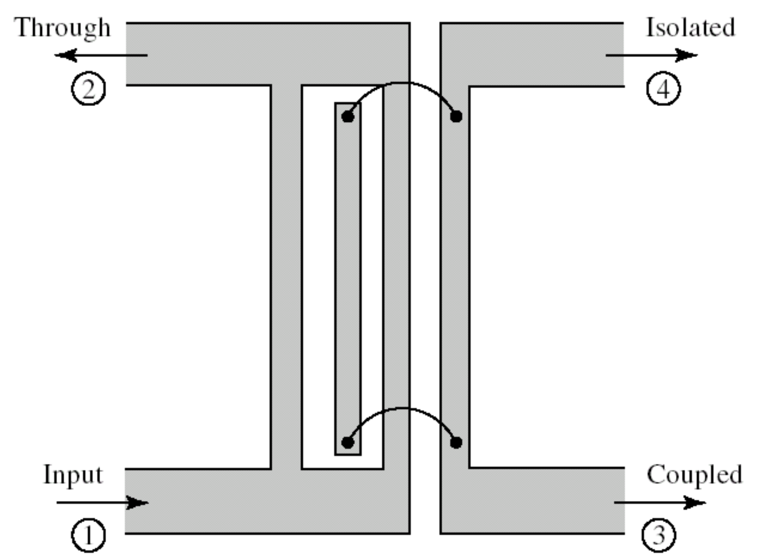
wide bandwidths: octave, decade is possible

the 3-dB Lange coupler is a quadrature hybrid

the design is based on that of a coupled-line directional coupler



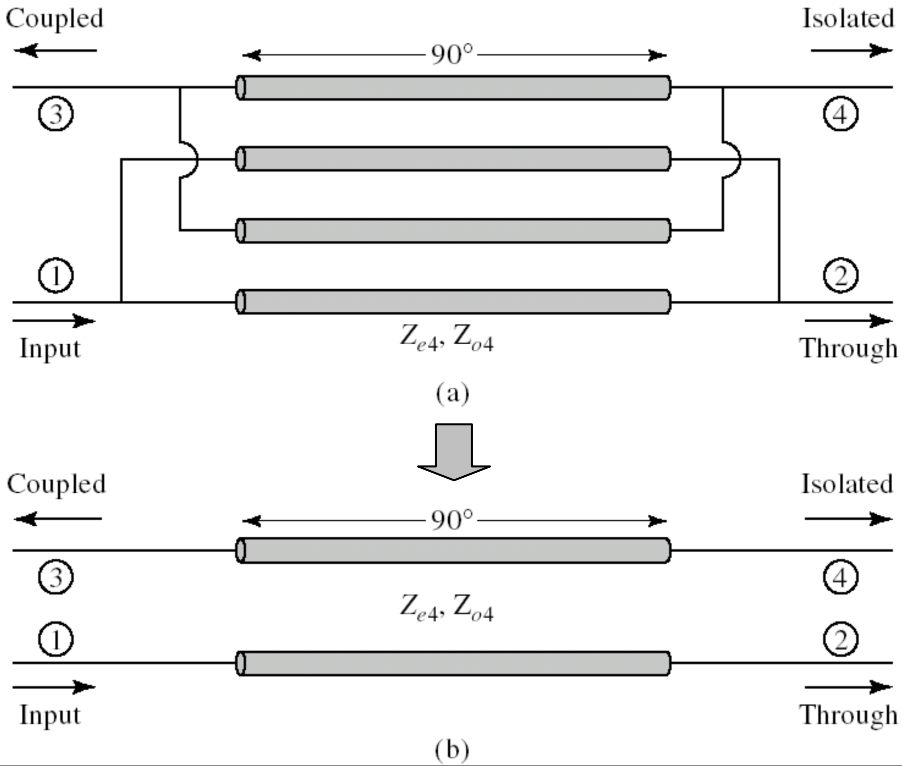
(a)



(b)

Lange Couplers: Equivalent Circuits

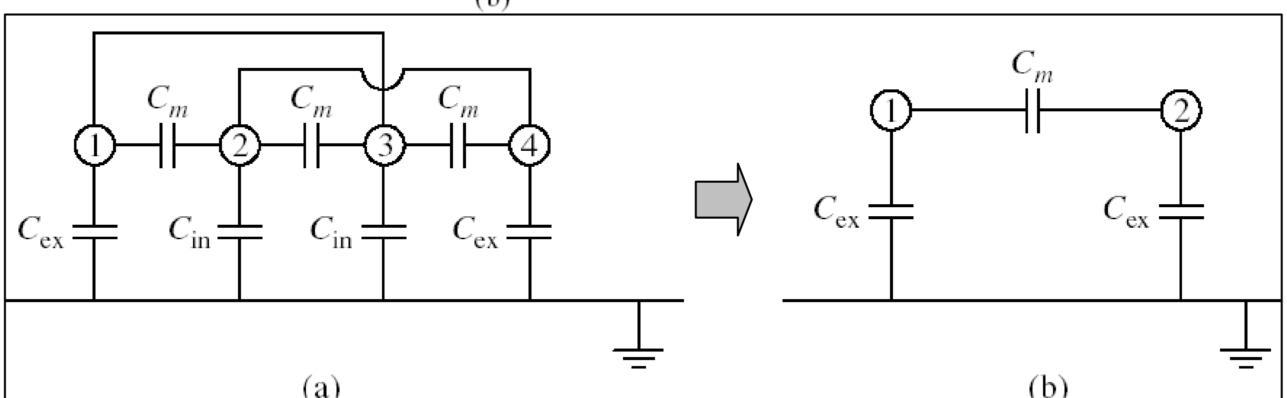
for 4 lines of same widths and spacings



$$Z_{e4} = Z_{0e}\frac{Z_{0o} + Z_{0e}}{3Z_{0o} + Z_{0e}}$$

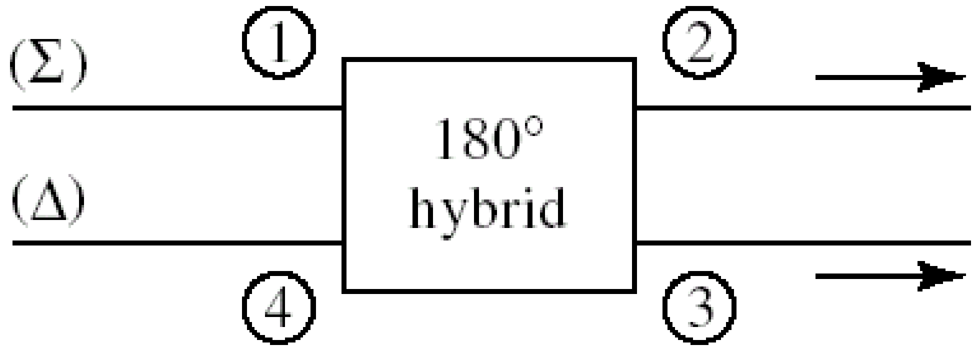
$$Z_{o4} = Z_{0o}\frac{Z_{0o} + Z_{0e}}{3Z_{0o} + Z_{0e}}$$

Z_{0e}, Z_{0o} - even and odd impedances of a pair of lines



180° Hybrids

$$\mathbf{S} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$



<u>hybrid</u>

input signal at port 1 is split equally into two in-phase signals at ports 2 and 3 (port 4 is isolated)

input signal at port 4 is split equally into two out-of-phase signals at ports 2 and 3 (port 1 is isolated)

<u>power combiner</u>

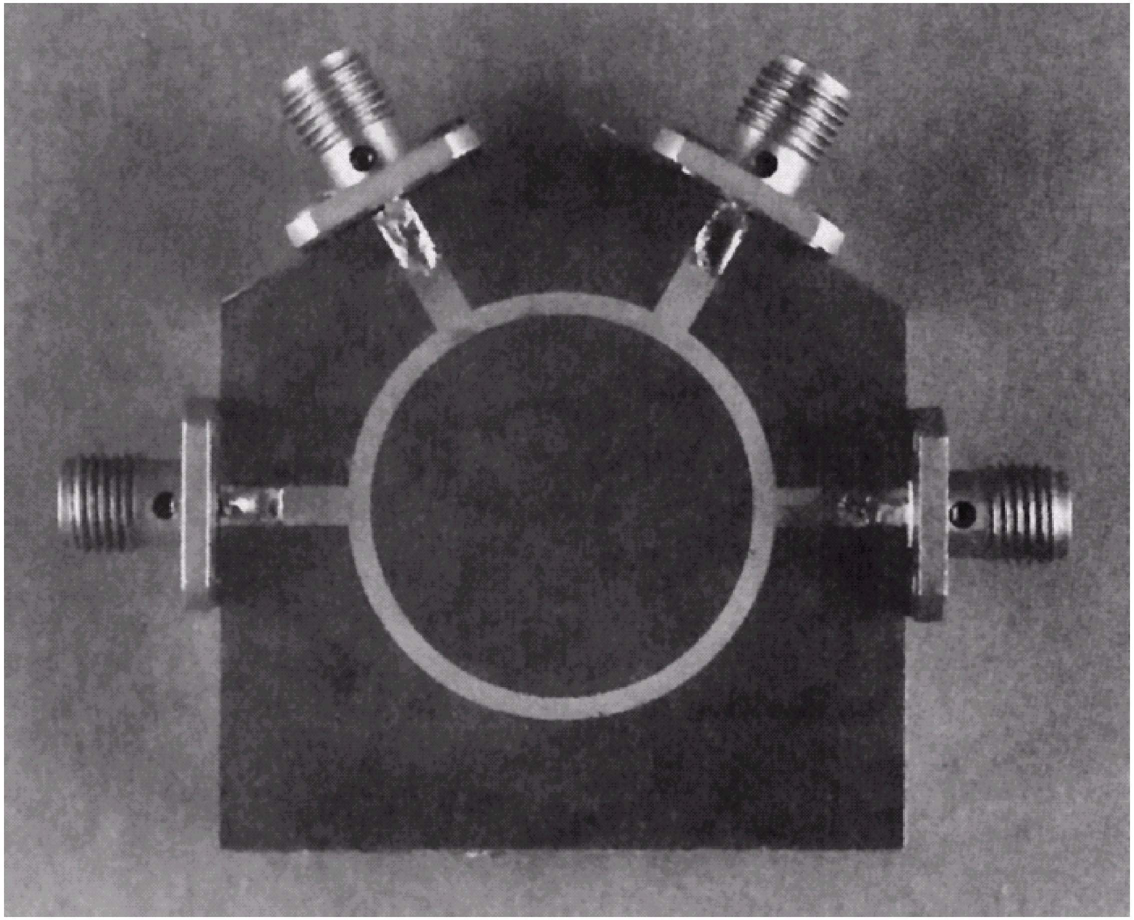
signals at ports 2 and 3 are added to produce the signal at port 1

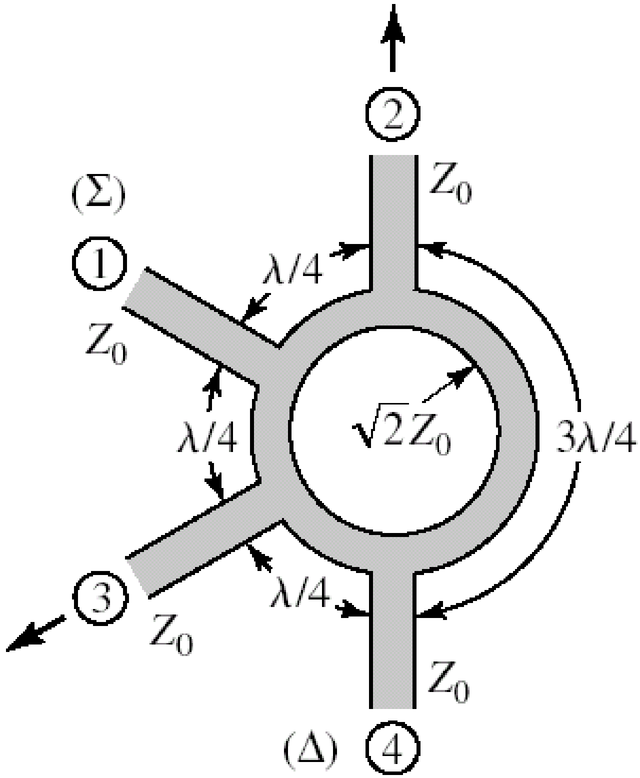
signals at ports 2 and 3 are subtracted to produce the signal at port 4

180° Ring (Rat-race) Hybrid

rigorous analysis through even/odd mode analysis – ring-line impedance must be $\sqrt{2}Z_0$

narrow-band performance



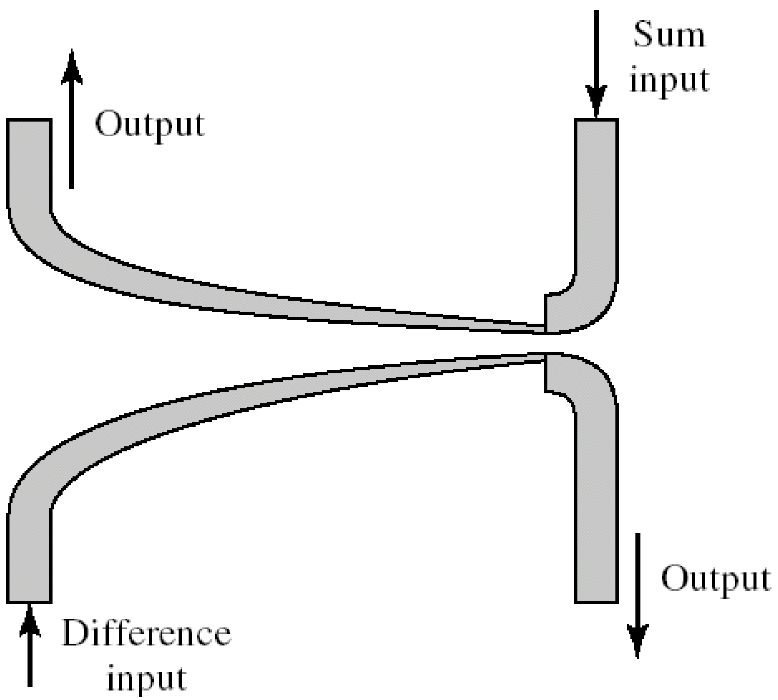


Tapered Coupled Line Hybrid

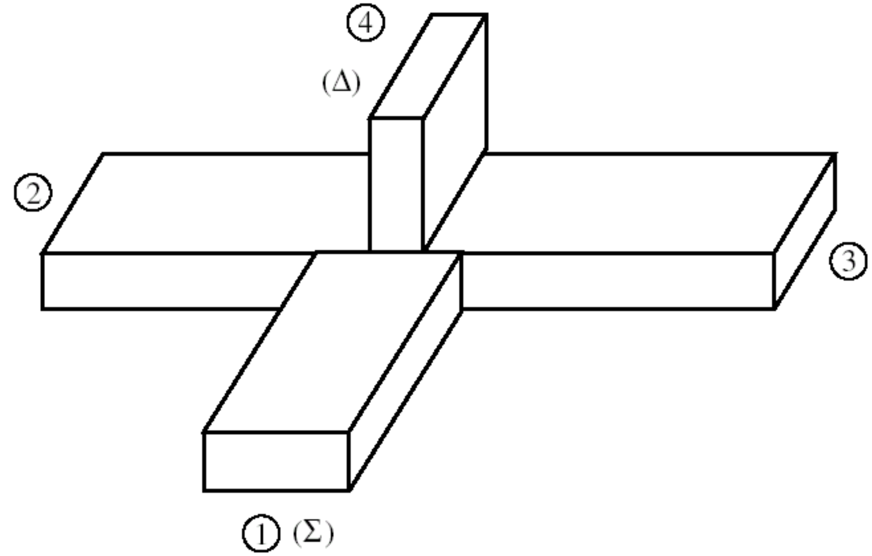
wide bandwidth (decade or more)

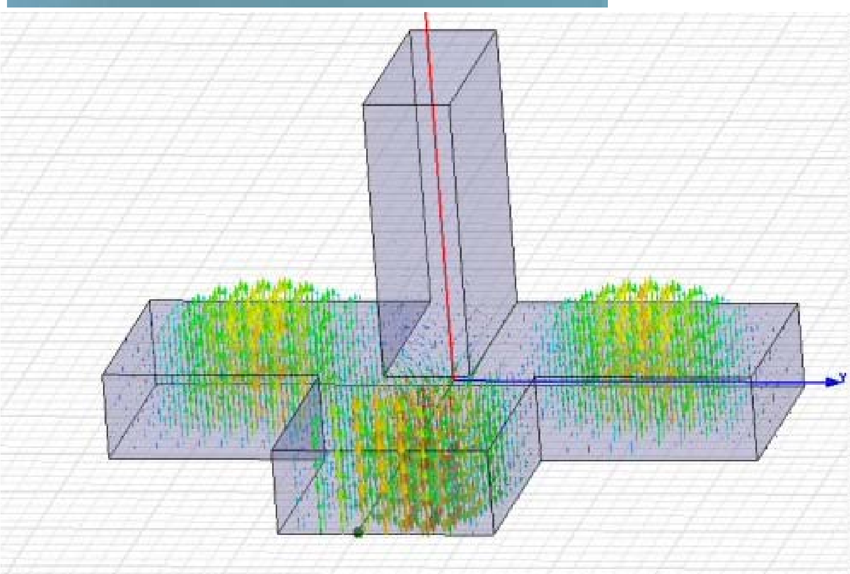
any power-division ratio can be achieved in principle

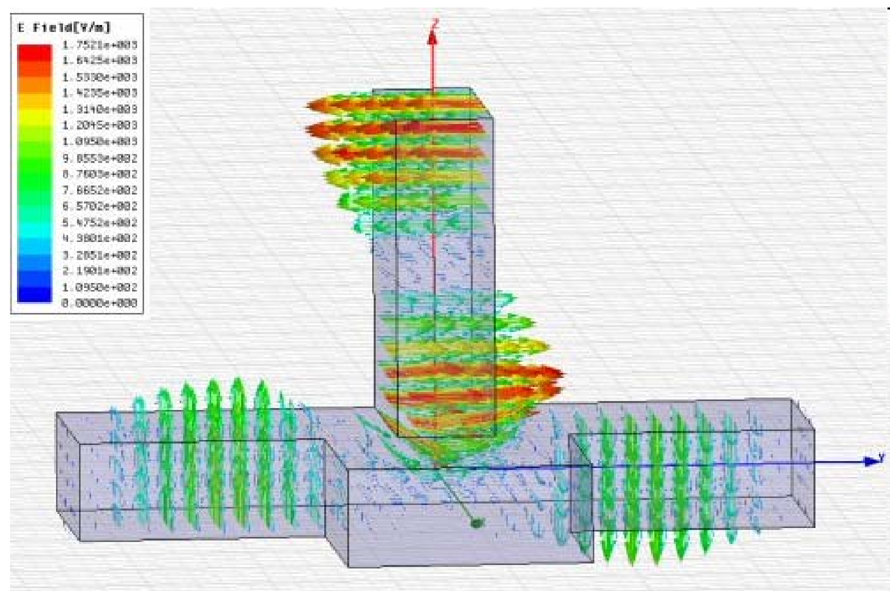
the hybrid is analyzed using even-odd mode analysis



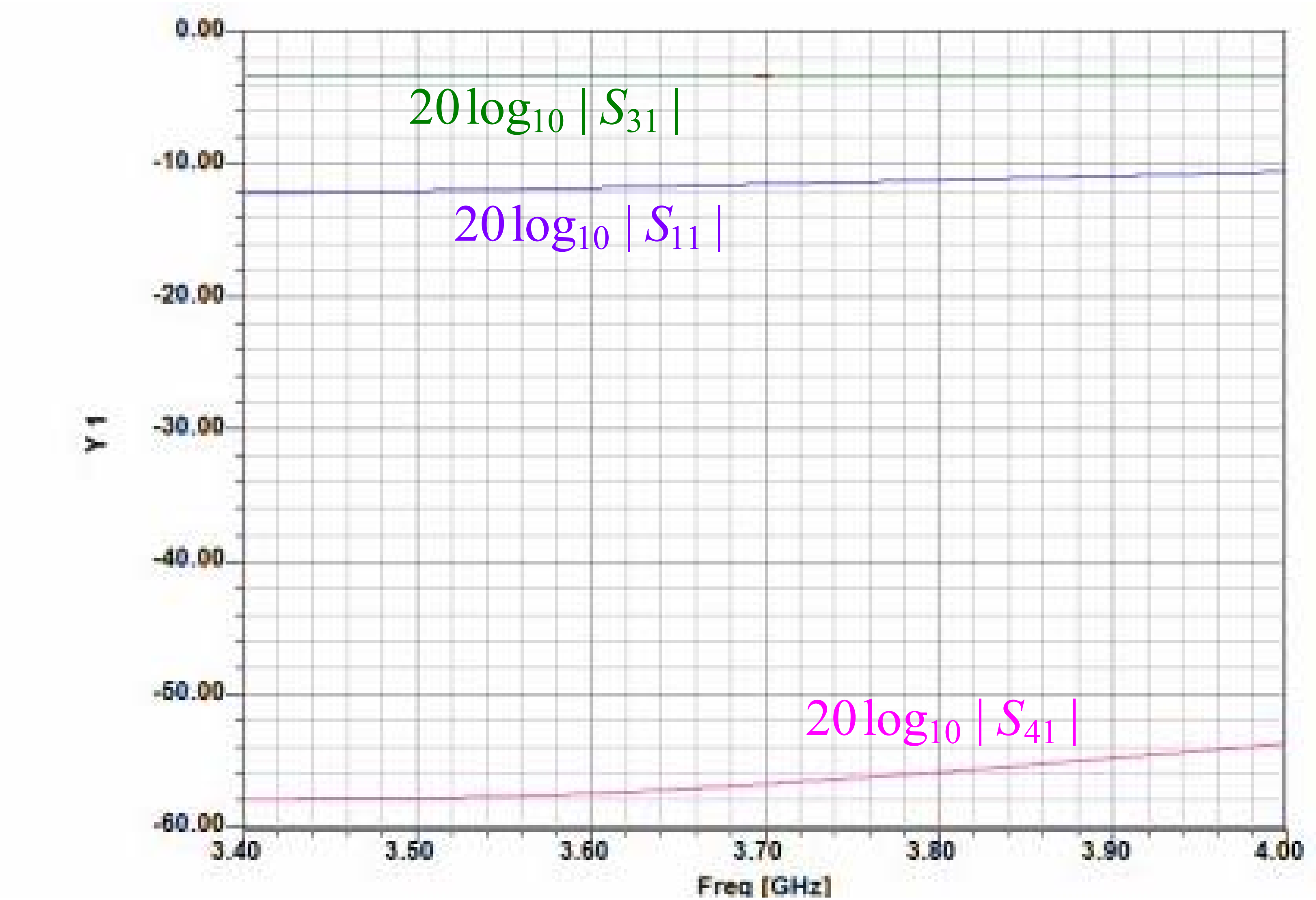
Waveguide Magic T







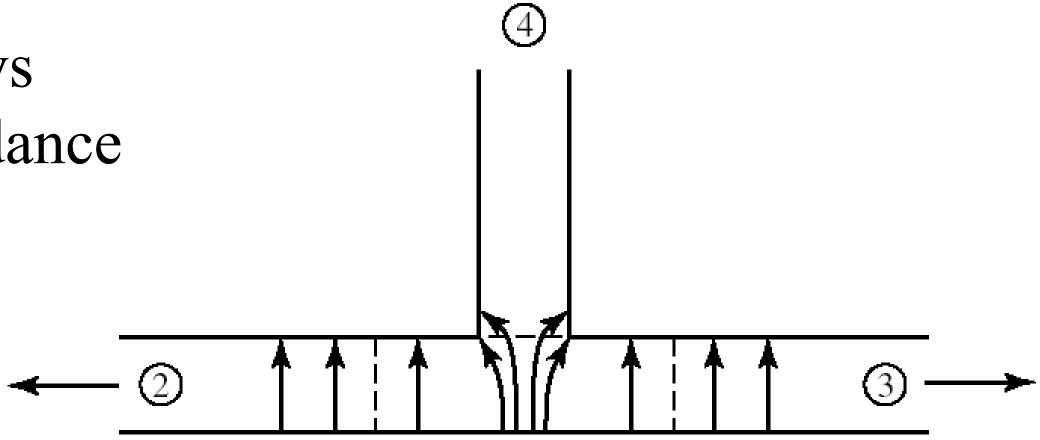
Waveguide Magic T (2)



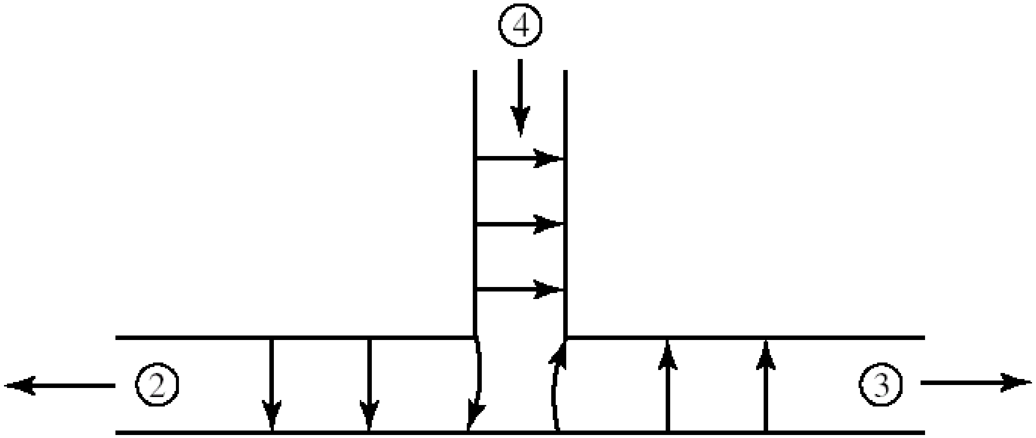
Waveguide Magic T (2)

principle of operation (TE_{10} mode)

tuning posts are always used to achieve impedance match

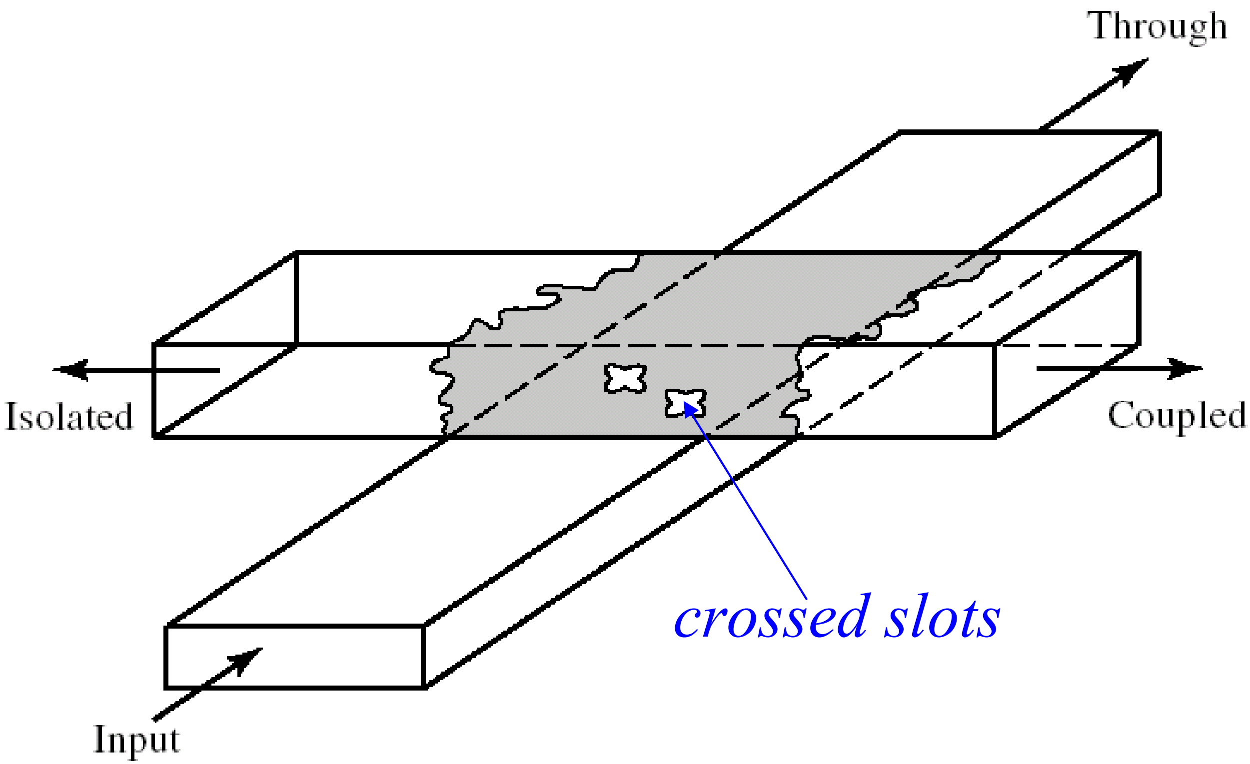


(a)



(b)

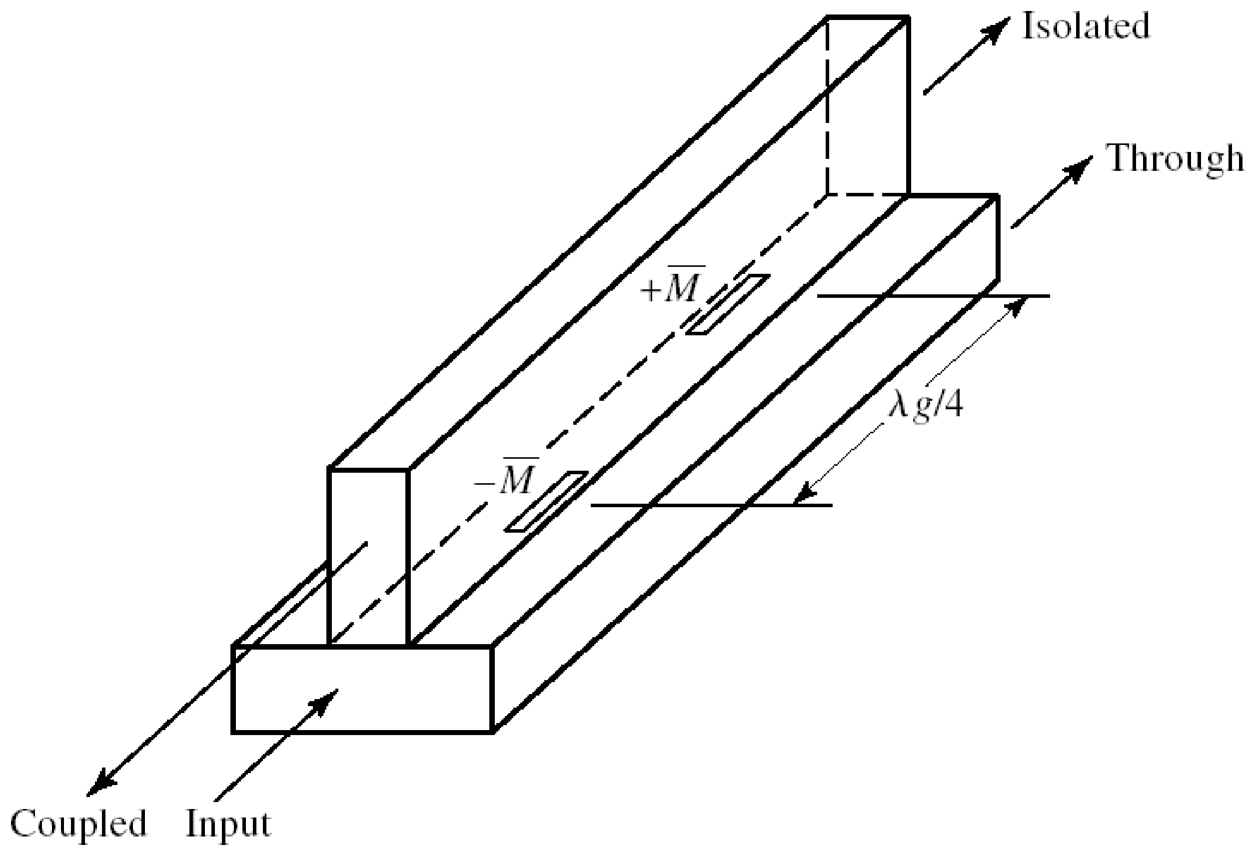
Moreno Crossed Guide Coupler



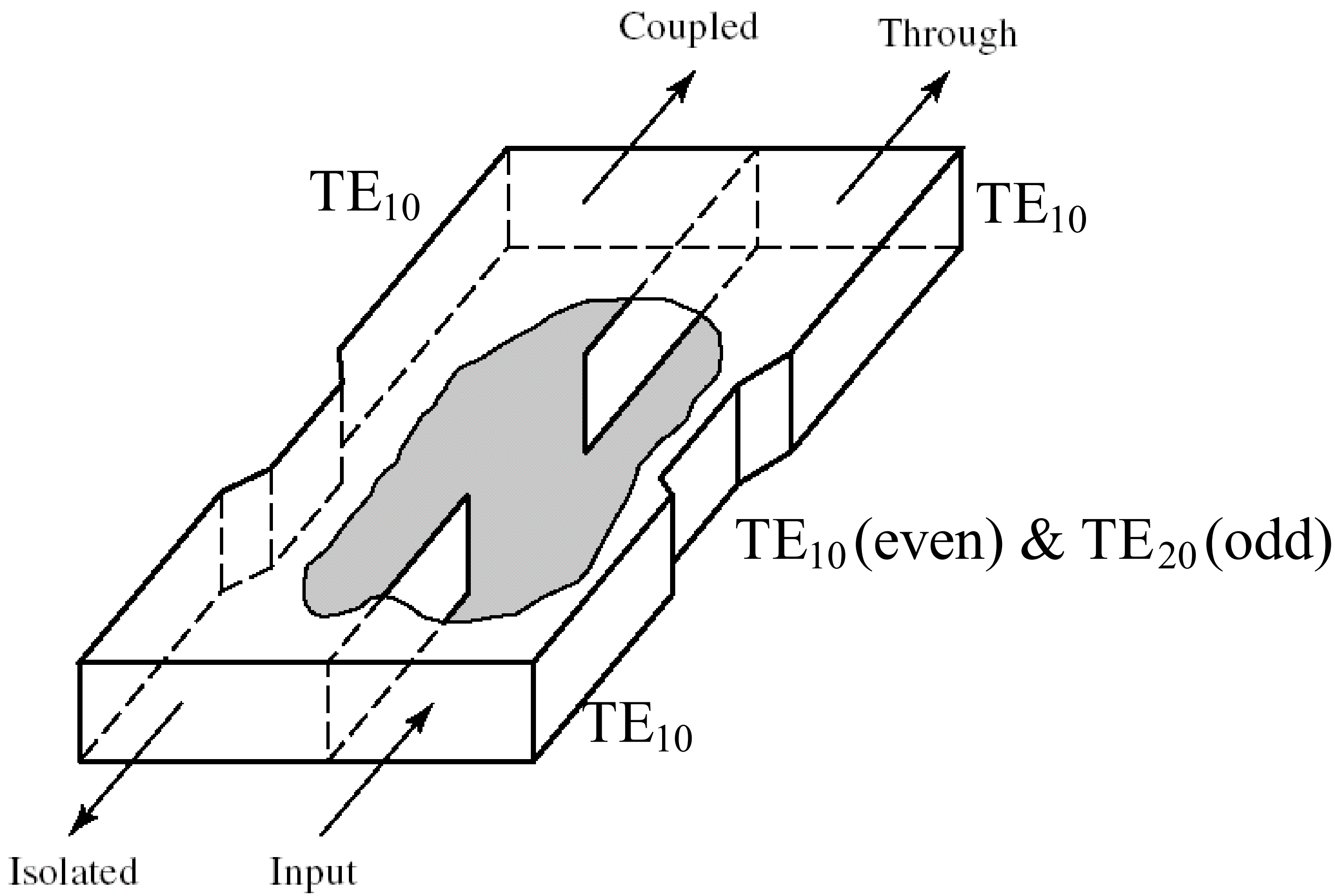
Schwinger Reversed-phase Coupler

slots are on opposite sides of the center line of the common wall

slots couple to H_z of opposite polarity



Riblet Slot Coupler



Coupled Planar Line Couplers (Multi-layer PCB designs)