6 Sequences and Series

6.1 Introduction

Sequences also called Progressions, are used to represent ordered lists of numbers. As the members of a sequence are in a definite order, so a correspondence can be established by matching them one by one with the numbers 1, 2, 3, 4, ... For example, if the sequence is 1, 4, 7, 10, ..., nth member, then such a correspondence can be set up as shown in the diagram below:

Position		the member of the sequence
1	$\longrightarrow$	1
2	$\longrightarrow$	4
3	$\longrightarrow$	7
4	$\longrightarrow$	10
$\vdots$		$\vdots$
n	$\longrightarrow$	nth member

Thus a sequence is a function whose domain is a subset of the set of natural numbers. A **sequence** is a special type of a function from a subset of N to $\mathbb{R}$ or C. Sometimes, the domain of a sequence is taken to be a subset of the set $\{0, 1, 2, 3, \ldots\}$, i.e., the set of non-negative integers. If all members of a sequence are real numbers, then it is called a **real sequence.**

Sequences are usually named with letters a, b, c etc., and n is used instead of x as a variable. If a natural number n belongs to the domain of a sequence a, the corresponding element in its range is denoted by a_n. For convenience, a special notation a_n is adopted for $a(n)$ and the symbol $\{a_n\}$ or $a_1, a_2, a_3, \ldots, a_n, \ldots$ is used to represent the sequence a. The elements in the range of the sequence $\{a_n\}$ are called its terms; that is, a_1 is the first term, a_2 the second term and a_n the nth term or the general term.

For example, the terms of the sequence $\{n+(-1)^n\}$ can be written by assigning to n, the values 1, 2, 3, ... If we denote the sequence by $\{b_n\}$, then

$$b_n = n + (-1)^n \text{ and we have}$$

$$b_1 = 1 + (-1)^1 = 1 - 1 = 0$$

$$\ldots 2 + 1 = 3$$

$$b_1 = 1 + (-1)^1 = 1 - 1 = 0$$
$$b_2 = 2 + (-1)^2 = 2 + 1 = 3$$
$$b_3 = 3 + (-1)^3 = 3 - 1 = 2$$
$$b_4 = 4 + (-1)^4 = 4 + 1 = 5 \text{ etc.}$$

If the domain of a sequence is a finite set, then the sequence is called a finite sequence otherwise, an infinite sequence.

Note: An infinite sequence has no last term.

Some examples of sequences are;

i) 1,4,9,.....,121 ii) 1,3,5,7,9,...., 21 iii) 1,2,4,...

iv) 1,3,7,15,31,... v) 1,6,20,56,... vi) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$

The sequences (i) and (ii) are finite whereas the sequences (iii) to (vi) are infinite.

6.2 Types of Sequences

If we are able to find a pattern from the given initial terms of a sequence, then we can deduce a rule or formula for the terms of the sequence.

We can find any term of the given sequence giving corresponding value to n in the nth / general term a_n of a sequence.

Example 1: Write first two, 21st and 26th terms of the sequence whose general term is $(-1)^{n+1}$.

Solution: Given that $a_n = (-1)^{n+1}$. For getting required terms, we put $n = 1, 2, 21$ and 26.

$$a_1 = (-1)^{1+1} = 1$$
$$a_2 = (-1)^{2+1} = -1$$
$$a_{21} = (-1)^{21+1} = 1$$
$$a_{26} = (-1)^{26+1} = -1$$

Example 2: Find the sequence if $a_n - a_{n-1} = n + 1$ and $a_4 = 14$

Solution: Putting $n = 2, 3, 4$ in

$a_n - a_{n-1} = n + 1$, we have

$a_2 - a_1 = 3$ (i)

$a_3 - a_2 = 4$ (ii)

$a_4 - a_3 = 5$ (iii)

From (iii), $a_3 = a_4 - 5$

$= 14 - 5 = 9$ $(\because a_4 = 14)$

From (ii), $a_2 = a_3 - 4$

$= 9 - 4 = 5$ $(\because a_3 = 9)$

And from (i), $a_1 = a_2 - 3$

$= 5 - 3 = 2$

Thus the sequence is $2, 5, 9, 14, 20, \ldots$

Note: $a_5 - a_4 = 6 \Rightarrow a_5 = a_4 + 6 = 14 + 6 = 20$

Exercise 6.1

1. Write the first four terms of the following sequences, if

i) $a_n = 2n - 3$ ii) $a_n = (-1)^n n^2$ iii) $a_n = (-1)^n (2n-3)$

iv) $a_n = 3n - 5$ v) $a_n = \dfrac{n}{2n+1}$ vi) $a_n = \dfrac{1}{2^n}$

vii) $a_n - a_{n-1} = n + 2, a_1 = 2$ viii) $a_n = n a_{n-1}, a_1 = 1$

ix) $a_n = (n+1) a_{n-1}, a_1 = 1$ x) $a_n = \dfrac{1}{a + (n-1)d}$

2. Find the indicated terms of the following sequences;

i) $2, 6, 11, 17, \ldots a_7$ ii) $1, 3, 12, 60, \ldots a_6$ iii) $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \ldots a_7$

iv) $1, 1, -3, 5, -7, 9, \ldots a_8$ v) $1, -3, 5, -7, 9, -11 \ldots a_8$

3. Find the next two terms of the following sequences;

i) $7, 9, 12, 16, \ldots$ ii) $1, 3, 7, 15, 31, \ldots$

iii) $-1, 2, 12, 40, \ldots$ iv) $1, -3, 5, -7, 9, -11, \ldots$

6.3 Arithmetic Progression (A.P)

A sequence $\{a_n\}$ is an Arithmetic Sequence or Arithmetic rogression (A.P), if $a_n - a_{n-1}$ is the same number for all $n \in N$ and $n > 1$. The difference $a_n - a_{n-1}, (n > 1)$ i.e., the difference of two consecutive terms of an A.P., is called the **common difference** and is usually denoted by d.

Rule for the nth term of an A.P.:

We know that $a_n - a_{n-1} = d \quad (n > 1)$.

which implies $a_n = a_{n-1} + d \quad (n > 1)$ ....(i)

Putting $n = 2, 3, 4, ...$ in (i) we get

$$a_2 = a_1 + d = a_1 + (2-1)d$$
$$a_3 = a_2 + d = (a_1 + d) + d$$
$$= a_1 + 2d = a_1 + (3-1)d$$
$$a_4 = a_3 + d = (a_1 + 2d) + d$$
$$= a_1 + 3d = a_1 + (4-1)d$$

Thus we conclude that

$$\boxed{a_n = a_1 + (n-1)d}$$

where a_1 is the first term of the sequence.

We have observed that

$$a_1 = a_1 + 0d = a_1 + (1-1)d$$
$$a_2 = a_1 + d = a_1 + (2-1)d$$
$$a_3 = a_2 + d = a_1 + (3-1)d$$
$$a_4 = a_3 + d = a_1 + (4-1)d$$

Thus $a_1, a_1 + d, a_1 + 2d, ..., a_1 + (n-1)d + ...$ is a general arithmetic sequence. with a_1, d as the first term and common difference respectively.

Note: $a_n = a_1 + (n-1)d$ is called the nth term or general term of the A.P.

Example 1: Find the general term and the eleventh term of the A.P. whose first term and the common difference are 2 and -3 respectively. Also write its first four terms.

Solution: Here, $a_1 = 2$, $d = -3$

We know that $a_n = a_1 + (n-1)d$,

so $\quad a_n = 2 + (n-1)(-3) = 2 - 3n + 3$

or $\quad a_n = 5 - 3n$

Thus the general term of the A.P. is $5 - 3n$. (i)

tting $n = 11$ in (i), we have

$$a_{11} = 5 - 3(11)$$
$$= 5 - 33 = -28$$

We can find a_2, a_3, a_4 by putting $n = 2,3,4$ in (i), that is,

$$a_2 = 5 - 3(2) = -1$$
$$a_3 = 5 - 3(3) = -4$$
$$a_4 = 5 - 3(4) = -7$$

Hence the first four terms of the sequence are: $2, -1, -4, -7$.

ample 2: If the 5th term of an A.P. is 13 and 17th term is 49, find a_n and a_{13}.

lution: Given $a_5 = 13$ and $a_{17} = 49$.

Putting $n = 5$ in $a_n = a_1 + (n-1)d$, we have

$$a_5 = a_1 + (5-1)d,$$
$$a_5 = a_1 + 4d$$

or $\quad 13 = a_1 + 4d \quad$ (i)

Also $a_{17} = a_1 + (17-1)d$

or $\quad 49 = a_1 + 16d$

or $\quad 49 = (a_1 + 4d) + 12d$

or $\quad 49 = 13 + 12d \quad$ (by (i))

$\Rightarrow \quad 12d = 36 \Rightarrow d = 3$

From (i), $a_1 = 13 - 4d = 13 - 4(3) = 1$.

Thus $\quad a_{13} = 1 + (13-1)3 = 37$ and

$$a_n = 1 + (n-1)3 = 3n - 2$$

ample 3: Find the number of terms in the A.P. if: $a_1 = 3, d = 7$ and $a_n = 59$.

lution: Using $a_n = a_1 + (n-1)d$, we have

$$59 = 3 + (n-1) \times 7 \qquad (\because a_n = 59, a_1 = 3 \text{ and } d = 7)$$

or $\quad 56 = (n-1) \times 7 \Rightarrow n - 1 = 8 \Rightarrow n = 9$

$b+c \quad c+a \quad a+b$

6.6 Word Problems on A.P.

Example 1: Tickets for a certain show were printed bearing numbers from 1 to 100. Odd number tickets were sold by receiving paisas equal to thrice of the number on the ticket while even number tickets were issued by receiving paisas equal to twice of the number on the ticket. How much amount was received by the issuing agency?

Solution: Let S_1 and S_2 be the amounts received for odd number and even number tickets respectively. Then

$$S_1 = 3[1+3+5+....+99] \text{ and } S_2 = 2[2+4+6+...+100]$$

Thus $S_1 + S_2 = 3\times\frac{50}{2}(1+99) + 2\times\frac{50}{2}(2+100)$, [$\because$ There are 50 terms in each series]

$$= 7500+5100 = 12600$$

Hence the total amount received by the issuing agency = 12600 paisas = Rs. 126

Example 2: A man repays his loan of Rs. 1120 by paying Rs. 15 in the first installment and then increases the payment by Rs. 10 every month. How long will it take to clear his loan?

Solution: It is given that the first installment (in Rs.) is 15 and the monthly increase in payment (in Rs.) is 10.

Here $a_1 = 15$ and $d = 10$

Let the time required (in months) to clear his loan be n. Then

$S_n = 1120$, that is,

$$1120 = \frac{n}{2}[2\times15 + (n-1)10] = \frac{n}{2}[30 + (n-1)10]$$

$$= \frac{n}{2}\times10[3 + (n-1)] = 5n(n+2)$$

or $\quad 224 = n(n+2) \Rightarrow n^2 + 2n - 224 = 0$

$$\Rightarrow \quad n = \frac{-2\pm\sqrt{4+896}}{2} = \frac{-2\pm\sqrt{900}}{2}$$

$$= \frac{-2\pm30}{2}$$

$$= 14, -16$$

But n can not be negative, so $n = 14$, that is, the time required to clear his loan is 14 months.

Example 3: A manufacturer of radio sets produced 625 units in the 4th year ... units in the 7th year. Assuming that production uniformly increases by a ... number every year, find

Arithmetic Mean (A.M)

A number A is said to be the A.M. between the two numbers a and b if a, A, b A.P. If d is the common difference of this A.P., then $A-a=d$ and $b-A=d$.

Thus $A-a=b-A$

or $2A=a+b$

$$\Rightarrow \quad A=\frac{a+b}{2}$$

Middle term of three consecutive terms in A.P. is the A.M. between the extreme terms.

In general, we can say that a_n is the A.M. between a_{n-1} and a_{n+1}, i.e.,

$$a_n=\frac{a_{n-1}+a_{n+1}}{2}$$

ple 1: Find three A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.

on: Let A_1, A_2, A_3 be three A.Ms between $\sqrt{2}$ and $3\sqrt{2}$. Then

$\sqrt{2}, A_1, A_2, A_3, 3\sqrt{2}$ are in A.P.

Here $a_1=\sqrt{2},\ a_5=3\sqrt{2}$

Using $a_n=a_1+(n-1)d$, we get

$$a_5=a_1+(5-1)d$$

or $$3\sqrt{2}=\sqrt{2}+4d$$

$$\Rightarrow \quad 3\sqrt{2}-\sqrt{2}=4d$$

$$\Rightarrow \quad d=\frac{2\sqrt{2}}{4}=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$

Now $$A_1=a_1+d=\sqrt{2}+\frac{1}{\sqrt{2}}=\frac{2+1}{\sqrt{2}}=\frac{3}{\sqrt{2}},$$

$$A_2=A_1+d=\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$$

$$A_3=A_2+d=2\sqrt{2}+\frac{1}{\sqrt{2}}=\frac{4+1}{\sqrt{2}}=\frac{5}{\sqrt{2}}$$

Therefore, $\frac{3}{\sqrt{2}}, 2\sqrt{2}, \frac{5}{\sqrt{2}}$ are three A.Ms between $\sqrt{2}$ and $3\sqrt{2}$.

6.8 Geometric Means

A number G is said to be a geometric mean (G.M.) between two numbers a and b if a, G, b are in G.P. Therefore

$$\frac{G}{a}=\frac{b}{G} \Rightarrow G^2=ab \Rightarrow G=\pm\sqrt{ab}$$

6.8.1 *n* Geometric Means Between two given numbers

The n numbers $G_1, G_2, G_3, \ldots, G_n$ are called n geometric means between a and b if $a, G_1, G_2, G_3, \ldots, G_n, b$ are in G.P.

Thus we have, $b = ar^{(n+2)-1}$ where r is the common ratio,

or $\quad ar^{n+1} = b$

$$\Rightarrow \quad r=\left(\frac{b}{a}\right)^{1/n+1}$$

Thus $\quad G_1 = ar = a\left(\frac{b}{a}\right)^{1/n+1}$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/n+1}$$

$$G_3 = ar^3 = a\left(\frac{b}{a}\right)^{3/n+1}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{n/n+1}$$

Note: $G_1.G_2.G_3 \ldots G_n = a^n\left(\frac{b}{a}\right)^{\frac{(1+2+3\ldots+n)}{n+1}} = a^n\left(\frac{b}{a}\right)^{\frac{n}{2}}$

and $\sqrt[n]{(G_1.G_2.G_3 \ldots G_n)} = a\left(\frac{b}{a}\right)^{1/2} = \sqrt{ab}$

$= G$, the geometric mean between a and b

Example 1: Find the geometric mean between 4 and 16.

Solution: Here $a = 4$, $b = 16$, therefore

$$G = \pm\sqrt{ab} = \pm\sqrt{4\times16}$$

$$= \pm\sqrt{64} = \pm 8$$

Example 2: Insert three G.Ms. between 2 and $\frac{1}{2}$.

Solution: Let G_1, G_2, G_3 be three G.Ms between 2 and $\frac{1}{2}$. Therefore 2, $G_1, G_2, G_3, \frac{1}{2}$ are in G.P. Here $a_1 = 2$, $a_5 = \frac{1}{2}$ and $n = 5$

Using $a_n = a_1 r^{n-1}$ we have,

$$a_5 = a_1 r^{5-1} \quad \text{i.e.,} \quad a_5 = a_1 r^4 \qquad \text{(i)}$$

Now substituting the values of a_5 and a_1 in (i) we have

$$\frac{1}{2} = 2r^4 \quad \text{or} \quad r^4 = \frac{1}{4} \qquad \text{(ii)}$$

Taking square root of (ii), we get,

$$r^2 = \pm\frac{1}{2}$$

So, we have, $r^2 = \frac{1}{2}$ or $r^2 = -\frac{1}{2} = \frac{i^2}{2}$ $\quad (\because \; -1 = i^2)$

$$\Rightarrow \qquad r = \pm\frac{1}{\sqrt{2}} \text{ or } r = \pm\frac{1}{\sqrt{2}}i$$

When $r = \frac{1}{\sqrt{2}}$, then $G_1 = 2\frac{1}{\sqrt{2}} = \sqrt{2}, G_2 = 2\left(\frac{1}{\sqrt{2}}\right)^2 = 1, G_3 = 2\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{\sqrt{2}}$

When $r = \frac{-1}{\sqrt{2}}$, then $G_1 = 2\left(\frac{-1}{\sqrt{2}}\right) = -\sqrt{2}, G_2 = 2\left(\frac{-1}{\sqrt{2}}\right)^2 = 1, G_3 = 2\left(\frac{-1}{\sqrt{2}}\right)^3 = -\frac{1}{\sqrt{2}}$

When $r = \frac{i}{\sqrt{2}}$, then $G_1 = 2\times\frac{i}{\sqrt{2}} = \sqrt{2}\,i, G_2 = 2\left(\frac{i}{\sqrt{2}}\right)^2 = -1, G_3 = 2\left(\frac{i}{\sqrt{2}}\right)^3 = -\frac{i}{\sqrt{2}}$

When $r = \frac{-i}{\sqrt{2}}$, then $G_1 = 2\left(\frac{-i}{\sqrt{2}}\right) = -\sqrt{2}\,i, G_2 = 2\left(\frac{-i}{\sqrt{2}}\right)^2 = -1, G_3 = 2\left(\frac{-i}{\sqrt{2}}\right)^3 = \frac{i}{\sqrt{2}}$

[redacted]

Example 3: If a, b, c and d are in G.P. show that $a+b, b+c$, $c+d$ are in G.P.

Solution: Since a, b, c are in G.P therefore,

$$ac = b^2 \qquad \text{(i)}$$

Also b, c, d are in G.P., so we have

Exercise 6.8

1. Find the sum of first 15 terms of the geometric sequence $1, \frac{1}{3}, \frac{1}{9}, \ldots$

2. Sum to n terms, the series

   i) $.2 + .22 + .222 + \ldots$ ii) $3 + 33 + 333 + \ldots$

3. Sum to n terms the series

   i) $1+(a+b)+(a^2+ab+b^2)+(a^3+a^2b+ab^2+b^3)+\ldots$

   ii) $r+(1+k)r^2+(1+k+k^2)r^3+\ldots$

4. Sum the series $2+(1-i)+\left(\frac{1}{i}\right)+\ldots$ to 8 terms.

5. Find the sums of the following infinite geometric series:

   i) $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\ldots$ ii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ iii) $\frac{9}{4}+\frac{3}{2}+1+\frac{2}{3}+\ldots$

   iv) $2+1+0.5+\ldots$ v) $4+2\sqrt{2}+2+\sqrt{2}+1+\ldots$ vi) $0.1+0.05+0.025+\ldots$

6. Find vulgar fractions equivalent to the following recurring decimals.

   i) $1.\dot{3}\dot{4}$ ii) $0.\dot{7}$ iii) $0.\dot{2}5\dot{9}$

   iv) $1.\dot{5}\dot{3}$ v) $0.\dot{1}5\dot{9}$ vi) $1.1\dot{4}\dot{7}$

7. Find the sum to infinity of the series: $r+(1+k)r^2+(1+k+k^2)r^3+\ldots$

   r and k being proper fractions.

8. If $y=\frac{x}{2}+\frac{1}{4}x^2+\frac{1}{8}x^3+\ldots$ and if $0<x<2$, then prove that $x=\frac{2y}{1+y}$

9. If $y=\frac{2}{3}x+\frac{4}{9}x^2+\frac{8}{27}x^3+\ldots$ and if $0<x<\frac{3}{2}$, then show that $x=\frac{3y}{2(1+y)}$

10. A ball is dropped from a height of 27 meters and it rebounds two-third of the distance it falls. If it continues to fall in the same way what distance will it travel before coming to rest?

11. What distance will a ball travel before coming to rest if it is dropped from a height of 75 meters and after each fall it rebounds $\frac{2}{5}$ of the distance it fell?

12. If $y=1+2x+4x^2+8x^3+\ldots$

    i) Show that $x=\frac{y-1}{2y}$

    ii) Find the interval in which the series is convergent.

13. If $y=1+\frac{x}{\ }+\frac{x^2}{\ }+\ldots$

Now taking a_1, the first term and d, the common difference of the corresponding A.P, we have,

$$a_1 + 3d = \frac{13}{2} \qquad \text{(i)}$$

and $$a_1 + 6d = \frac{25}{2} \qquad \text{(ii)}$$

Subtracting (i) from (ii), gives

$$3d = \frac{25}{2} - \frac{13}{2} = 6 \Rightarrow d = 2$$

From (i), we get

$$a_1 = \frac{13}{2} - 3d = \frac{13}{2} - 6 = \frac{1}{2}$$

Thus a_2 of the A.P. $= a_1 + d = \frac{1}{2} + 2 = \frac{5}{2}$

And a_3 of the A.P. $= a_1 + 2d = \frac{1}{2} + 2(2)$

$$= \frac{1}{2} + 4 = \frac{9}{2}$$

Hence the required H.P. is $\frac{2}{1}, \frac{2}{5}, \frac{2}{9}, \frac{2}{13}, \ldots$

6.12.1 Harmonic Mean: A number H is said to be the harmonic mean (H.M) between two numbers a and b if a, H, b are in H.P.

Let a, b be the two numbers and H be their H.M. Then $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

therefore, $$\frac{1}{H} = \frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{\frac{b+a}{ab}}{2} = \frac{a+b}{2ab}$$

and $$H = \frac{2ab}{a+b}$$

For example, H.M. between 3 and 7 is

$$\frac{2\times3\times7}{3+7} = \frac{2\times21}{10} = \frac{21}{5}$$

6.12.2 n Harmonic Means between two numbers

$H_1, H_2, H_3, \ldots, H_n$ are called n harmonic means (H.Ms) between a and b if

6.12.2 n Harmonic Means between two numbers

$H_1, H_2, H_3, \ldots, H_n$ are called n harmonic means (H.Ms) between a and b if $a, H_1, H_2, H_3, \ldots, H_n, b$ are in H.P. If we want to insert n H.Ms. between a and b, we first find n A.Ms. $A_1, A_2, \ldots, A_n$ between $\frac{1}{a}$ and $\frac{1}{b}$, then take their reciprocals to get n H.Ms. between a and b, that is, $\frac{1}{A_1}, \frac{1}{A_2}, \ldots, \frac{1}{A_n}$ will be the required n H.Ms. between a and b.

Example 3: Find three harmonic means between $\frac{1}{5}$ and $\frac{1}{17}$

Solution: Let A_1, A_2, A_3 be three A.Ms. between 5 and 17, that is,

$5, A_1, A_2, A_3, 17$ are in A.P.

Using $a_n = a_1 + (n-1)d$, we get,

$$17 = 5 + (5-1)d \quad (\because a_5 = 17 \text{ and } a_1 = 5)$$

$$4d = 12$$

$$\Rightarrow \quad d = 3$$

Thus $A_1 = 5 + 3 = 8$, $A_2 = 5 + 2(3) = 11$ and $A_3 = 5 + 3(3) = 14$

Hence $\frac{1}{8}, \frac{1}{11}, \frac{1}{14}$ are the required harmonic means.

Example 4: Find n H.Ms between a and b

Solution: Let $A_1, A_2, A_3, \ldots, A_n$ be n A.Ms between $\frac{1}{a}$ and $\frac{1}{b}$.

Then $\frac{1}{a}, A_1, A_2, A_3, \ldots, A_n, \frac{1}{b}$ are in A.P.

Using $a_n = a_1 + (n-1)d$, we get,

$$\frac{1}{b} = \frac{1}{a} + (n+2-1)d$$

or
$$(n+1)d = \frac{1}{b} - \frac{1}{a} \Rightarrow d = \frac{a-b}{ab(n+1)}$$

Thus
$$A_1 = \frac{1}{a} + d = \frac{1}{a} + \frac{a-b}{ab(n+1)} = \frac{b(n+1)+(a-b)}{ab(n+1)} = \frac{nb+a}{ab(n+1)}$$

$$A_2 = \frac{1}{a} + 2d = \frac{1}{a} + 2\left(\frac{a-b}{ab(n+1)}\right) = \frac{b(n+1)+2(a-b)}{ab(n+1)} = \frac{(n-1)b+2a}{ab(n+1)}$$