***Chapter***

**3**

***Matrices and Determinants***

**Matrix**

A rectangular array of numbers enclosed by a pair of brackets is called a *matrix*. In rectangular array *mn* of numbers in the form of m horizontal lines ( called **rows**) and n vertical lines (called **columns**).

Matrix usually use capital letters such as  etc., to represent the matrices and small letters such as a,

 etc., to indicate the entries of the matrices.

**Examples**



**Rows:** In rectangular array the horizontal lines of numbers are called rows. The number of rows is denoted by n.

**Columns:** The vertical lines of numbers are called columns. The number of columns is denoted by m.

**Elements of Matrix:** The numbers used in rows or columns are said to be the entries or elements of the matrix.

**Order of a matrix:**

If a matrix has m rows and n columns is called a matrix of order ***m*** by ***n***, written as  matrix. **Example**

 is a matrix having two rows and 3 columns. Its order is 

and it has 6 elements



**TYPES OF MATRICES**

**Row Matrix:**A matrix having only one row is called a row matrix or a row vector. For example, is a row matrix of order.

**Column Matrix:** A matrix having only one column is called a column matrix or a column vector.

For example, 

**Rectangular Matrix:** If the matrix in which the number of rows is not equal to the number of columns is said to be a rectangular matrix, that is, , then the matrix is called a rectangular matrix of order . For example are rectangular matrices of orders  respectively.

**Square Matrix:** If , then the matrix of order is said to be a square matrix of order *n* or *m.* i.e., the matrix which has the same number of rows and columns is called a square matrix. For example;

 are square matrices of orders 1, 2 and 3 respectively.

**Diagonal Matrix:** A square matrix is called a diagonal matrix if all the elements except those in the leading diagonal are zero, i.e. for and for .

*For example*, the matrix A =  is a diagonal matrix, and is denoted by .

**Scalar Matrix:** A square matrix in which every non-diagonal element is zero and all diagonal elements are equal, is known as scalar matrix.

*For example,* the matrices A =  and

B =  are scalar matrices of order 2 and 3 respectively.

**Unit Matrix:** A square matrix in which every non-diagonal element is zero and every diagonal element is 1, is called a unit or identity matrix.

A unit matrix of order n is denoted by . For example  and  are unit matrices of order 2 and 3 respectively.

**Trace of a Matrix:** The sum of the diagonal elements of a square matrix *A* is called the trace of *A* and is denoted by *tr(A)*. For example, if A = 

then tr(A) =4 + 1 + (– 8) = – 3

**Equal Matrices:** Two matrices *A* and *B* are said to be equal if they are of same order and all the corresponding elements are equal. It is written as

A = B.

For example 

are equal matrices, whereas  and  are not equal, because their orders are not same.

**ALGEBRA OF MATRICES**:

**1. Addition:** Two matrices are confirm for addition if both have same order. Let A and B be two matrices each of order . Then the sum of matrices is denoted by A + B. The new matrix, say C = A + B is of  and is obtained by adding the corresponding elements of A and B.

For example, if  and 

then 



whereas the addition of  and is not defined because given two matrices are not of same order.

**Addition Properties**

1. ***Addition of matrices is commutative.***

If A and B are two matrices of the

same order, then *A + B = B + A.*

1. ***Addition of matrices is associative.*** If A,

B and C are three matrices of the same

order, then *(A + B) + C = A + (B + C).*

1. ***Additive identity****.* If 0 is the zero matrix

of the same order as that of the matrix A,

then *A + O = A = O + A.*

**Remark:**

Null matrix or zero matrix is operates same in addition of matrices like as the number zero does in addition of numbers.

**2. Subtraction:** The subtraction of matrices is possible if both matrices have same order. Let *A* and *B* be two matrices of the same order. To find A – B, we subtract each element of B from the corresponding element of A.

For Example, , 





**3. Multiplication:** Two matrices *A* and *B* are confirmable for multiplication if the number of columns in *A* is same as the number of rows in *B.* For example, if and  are two matrices of order  and  respectively, then their product AB is of order  and *BA* is of order 



Similarly



**Multiplication Properties**

1. ***Distributive Property:***

if *A, B, C* are , , matrices

respectively, then *A(B + C) = AB + AC.*

1. ***Associative Property:***

If A, B, C are matrices of order , ,

and  respectively, then *(AB)C = A(BC)*

1. ***Multiplicative Identity:***

If A is a square matrix

of order n, the *A In = In A = A.,* where *In*is

identity matrix of order n

1. If *AB = 0* (null matrix) does not mean that *A = 0* or *B = 0* or both *A = B = 0*.

For example,

if  and , then 

It can be easily observed both matrices A

and B are non-zero but their product is a null

matrix.

**4. Multiplication of a Matrix by a Scalar:** Let  be an matrix and k be any scalar. Then the matrix obtained by multiplying each element of *A* by *k* is called the scalar multiple of *A* by *k* and is denoted by k*A*. Thus, if , then .

For example, if

then



**Properties of Scalar Multiplication**

1. If *A* and *B* are two matrices of the same order and *k* be a scalar, then *k(A + B) = kA + kb*
2. If *k1* and *k2* are two scalars and *A* is a matrix, then *(k1+k2) A = k1A + k2A.*
3. If *k1* and *k2* are two scalars and *A* is a matrix, *(k1k2)A = k1(k2A) = k2 (k1A)*
4. If *A* is any matrix, then *1A = A.*

**TRANSPOSE OF A MATRIX:** let A be aorder matrix. Then, the matrix obtained by interchanging the rows and columns of *A* is called the transpose of *A*, and is denoted by *A’* or *At.* Thus,

1. If order of *A* is , then, the order of *At* is .
2. *(i, j)th* element of *A* = *(j, i)th* element of *At.*

For example, if ,



**Determinant of a 2×2 matrix:** We can associate with every square matrix *A* over  or C, a number *|A|*, known as the determinant of the matrix *A*. The determinant of a matrix is denoted by enclosing its square array between vertical bars instead of brackets.

For example, if,  then the determinant of A is .

Its value is defined to be the real number *ad – bc,* that is



**Singular and Non-Singular Matrices:**

A square matrix *A* is singular if, otherwise it is a nonsingular matrix.



is a singular matrix and



is a non-singular matrix.

**Adjoint of a 2×2 Matrix:** The adjoint of the matrix is denoted by *adj A* and is defined as: 

**Inverse of a square matrix:**

Let A be any square matrix of order *n*. The inverse of *A* denoted by A-1 is determined by the formula:



It may be noted that  =  = *I*,

**Properties of the inverse of a Matrix**

1. A square matrix is invertible if and only if it is non-singular.
2. The inverse of transpose of a matrix is the transpose of its inverse, i.e., 
3. The inverse of the inverse is the original matrix, i.e.  = *A*
4. If *A* and *B* are two invertible matrices of the same order, then *AB* is also invertible and 
5. Let *A, B, C* be square matrices of same order n, If A is a non-singular matrix, then
6. *AB = AC B = C* (Left cancellation law)
7. *BA = CA B = C* (Right cancellation law)
8. If *A* is a non-singular matrix such that A is symmetric thenis also symmetric.

**Remarks**

(i) These cancellation laws hold only

when matrices are non-singular

(ii)If A is a non-singular matrix, then.

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***Exercise 3.1***

**Q. No. 1:**

**If** **and** **, then show that.**

i) ii) 

**Solution**☺

**(i)**









**(ii)** 













Hence *L.H.S. = R.H.S*

**Q. No. 2: :** **Find x and y if.**

i) 

ii)

**Solution**☺

**(i)**



“Two matrices are said to be equal if

both have same order and sane

corresponding elements”.



**(ii)**



“Two matrix are said to be equal if both have

same order and sane corresponding elements”.





Using the value y from (1) in (2)









Putting value of x in (1)



**Q. No. 3:**

**If** **and** **, find the following matrices.**

**i)**  ii) 

**Solution**☺



















**Q. No. 5:**

**Find x and y if** 

**Solution**☺









**Q. No. 6:**

**If  show that**

**i) **

**ii) **

**iii) **

**Solution**☺

**(i)** 

Let 

L.H.S = 









= R.H.S.

**(ii) **

Let 



L.H.S.











= R.H.S.

**(ii)**









**Q. No. 7:**

**If**  and **show** **that** 

**Solution**☺



L.H.S



















= R.H.S

**Q. No. 8:**

**If**  and , find the values of and .

**Solution**☺



 

As 







**Q. No. 9:**

**If**  and  find the values of and .

**Solution**☺



 







**Q. No. 10:**

**If** **and** **, then show that** 

**Solution**☺















Hence 

**Q. No.11:**

**Find**  **if** 

**Solution**☺













**Q. No.11:**

**Find the matrix X if**

**i)** 

**ii) **

**Solution**☺

**(i)**

Let , 

We can write the equation in the form as





Now,





Putting the values in equation (1) we get



**ii)** 

Let , 

We can write the equation in the form as





Now, 



Putting the values in equation (1) we get



**Q. No.12:**

**Find the matrix  if**

**i) **

**ii) **

**Solution**☺

**(i) **

Let 





“Two matrixes of same order are equal if their

corresponding elements are equal”

 

 

Adding (1) and (2)



Putting 



Adding (3) and (4)  putting 



Hence 

**(ii)**



We can write the equation in the form as



















 or 

Multiply eq (2) by 2 and adding into eq (1)





Put in eq. (1)

  

 or 

Multiply eq (4) by 2 and adding into eq (3)



, using in eq. (3)

  

 or 

Multiply eq (6) by 2 and adding into eq (5)



, using in eq. (5)

  

 

**Q. No.12:**

**Show that**  

**Solution:**

L.H.S







 

♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦

***Exercise 3.2***

**Q. No. 1:**

**If , then show that**

**i)  ii) **

**Solution**☺

**L.H.S =**







= R.H.S

**L.H.S = **







= R.H.S

**Q. No. 2: Find the inverse of the following matrices.**

**i)  ii) **

**iii)  iv) **

**Solution**☺

Let

****

****

****

As A is non-singular, so  exists





**(ii)**

**Let **

****

****

****

Since A is non-singular, so exists





**(iii)**

Let ****

****

****

****

As A is non-singular, so  exist





**(iv)**

Let ****

****

****

****

Since, i.e., matrix is singular.

Hence  doesn’t exist.

**Q. No. 3: Solve the following system of linear equations.**

**i)  ii) **

**iii) **

**Solution**☺

****

****





**=**

Since A is non-singular, so  exists



 

Put value if  in equation (1)







Hence 



**(ii)**

****

****





**=**

Since A is non-singular, so  exists

  

Put value if  in equation (1)







Hence 



**(iii)**

****

****





**=**

Since A is non-singular, so  exists



 

Using the value of  in equation (1), we get







Hence





**Q. No. 4:**

**If ,  and , then find**

**i)  ii) **

**iii)  iv) **

**Solution**☺

**(i) **

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**(ii)**

****

****

****

**(iii)**

****

****





**(iv)**

****

****

****



**Q. No. 5:**

**If ,  and  then show that**

**i) **

**ii) **

**Solution**☺

****

****







****







It is clear form Eqs. (1) and (2) *L.H.S = R.H.S.*

**(ii)**

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****

****



****

****



It is clear from (1) and (2) *L.H.S. = R.H.S.*

**Q. No. 5:**

**If A and B are square matrices of the same order, then explain why in general:**

**i) **

**ii) **

**iii) **

**Solution**☺

**(i) **



In matrix multiplication,





Hence ****

**(ii)**

****





In matrix multiplication, 



Hence ****

**(iii)**

****



In matrix multiplication, 



Hence ****

**Q NO.7: If , then find **

**Solution: ** 

****

****

****

**Q No.8: Solve the following matrix equation for X:**

**i)**



**ii)**



**Solution:**

**i)**









**ii)**









**Q No.9: Solve the following matrix equation for A:**

i) 

ii) 

**Solution:**

**i)**











Now 

B is a non-singular, so  exists



Using value of in eq. (1)



**ii)**







Now 

B is a non-singular, so  exists





Putting value of  in eq. (1)



♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦