Induced Electric Field

Let us place a copper ring of radius *r* in a uniform external magnetic field, as in Fig. *a*. The field—neglecting fringing—fills a cylindrical volume of radius *R.* Suppose that we increase the strength of this field at a steady rate, perhaps by increasing—in an appropriate way—the current in the windings of the electromagnet that produces the field. The magnetic flux through the ring will then change at a steady rate and—by Faraday's law—an induced emf and thus an induced current will appear in the ring. From Lenz's law we can deduce that the direction of the induced current is counter clockwise in Fig. *a*. If there is a current in the copper ring, an electric field must be present along the ring because an electric field is needed to do the work of moving the conduction electrons. Moreover, the electric field must have been produced by the changing magnetic flux. This **induced electric field** is just as real as an electric field produced by static charges; either field will exert a force on a particle of charge *q*0. By this line of reasoning, we are led to a useful and informative restatement of Faraday's law of induction:

A changing magnetic field produces an electric field.

The striking feature of this statement is that the electric field is induced even if there is no copper ring. Thus, the electric field would appear even if the changing magnetic field were in a vacuum. To fix these ideas, consider Fig. *b,* which is just like Fig. *a* except the copper ring has been replaced by a hypothetical circular path of radius *r*. We assume, as previously, that the magnetic field B is increasing in magnitude at a constant rate *dB*/*dt*. The electric field induced at various points around the circular path must—from the symmetry—be tangent to the circle, as Fig. *b* shows.* Hence, the circular path is an electric field line. There is nothing special about the circle of radius *r*, so the electric field lines produced by the changing magnetic field must be a set of concentric circles, as in Fig. *c*.

As long as the magnetic field is *increasing* with time, the electric field represented by the circular field lines in Fig. *c* will be present. If the magnetic field remains *constant* with time, there will be no induced electric field and thus no electric field lines. If the magnetic field is *decreasing* with time (at a constant rate), the electric field lines will still be concentric circles as in Fig. *c*, but they will now have the opposite direction. All this is what we have in mind when we say "A changing magnetic field produces an electric field."

A Reformulation of Faraday's Law

Consider a particle of charge q_0 moving around the circular path of Fig. *b*. The work *W* done on it in one revolution by the induced electric field is $W = \xi q_0$, where ξ is the induced emf—

that is, the work done per unit charge in moving the test charge around the path. From another point of view, the work is

$$W = \int \vec{F} \cdot d\vec{s} = (q_0 E)(2\pi r),$$

where $q0E$ is the magnitude of the force acting on the test charge and $2\pi r$ is the distance over which that force acts. Setting these two expressions for W equal to each other and cancelling $q0$, we find that

$$\mathscr{E} = 2\pi r E.$$

The work done on a particle of charge $q0$ moving along any closed path:

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}.$$

(The loop on each integral sign indicates that the integral is to be taken around the closed path.) Substituting ξq_o for W, we find that

$$\mathscr{E} = \oint \vec{E} \cdot d\vec{s}.$$

Meaning of emf. With above eq., we can expand the meaning of induced emf. Up to this point, induced emf has meant the work per unit charge done in maintaining current due to a changing magnetic flux, or it has meant the work done per unit charge on a charged particle that moves around a closed path in a changing magnetic flux. However, with Fig. *b* and above eq., an induced emf can exist without the need of a current or particle: An induced emf is the sum—via integration—of quantities $\bar{E}.ds$ around a closed path, where is the electric field induced by a changing magnetic flux and *ds* is a differential length vector along the path.

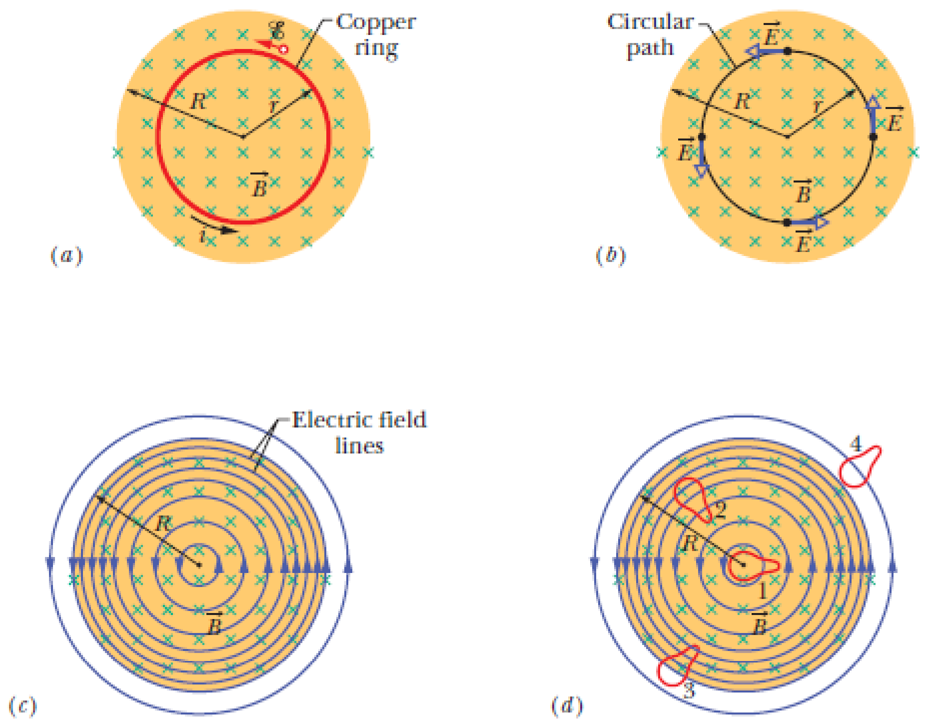
If we combine above equation with Faraday's law ($\xi = -d\phi_B/dt$), we can rewrite Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law).}$$

This equation says simply that a changing magnetic field induces an electric field.

The changing magnetic field appears on the right side of this equation, the electric field on the left.

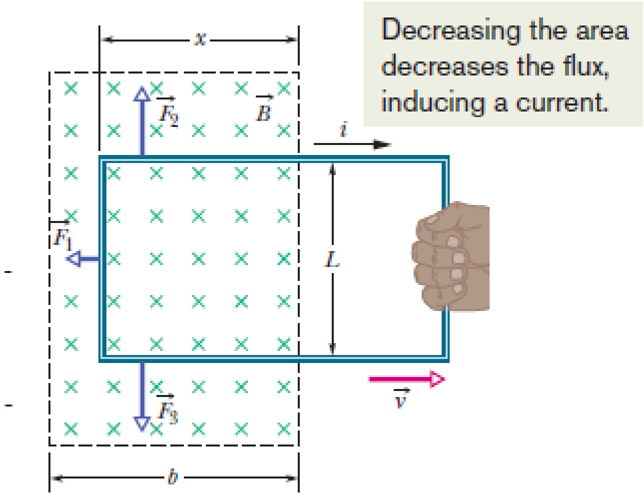
Faraday's law in the form of above equationcan be applied to *any* closed path that can be drawn in a changing magnetic field. Figure *d*, for example, shows four such paths, all having the same shape and area but located in different positions in the changing field. The induced emfs ξ $(= \oint \bar{E}.\, ds)$ for paths 1 and 2 are equal because these paths lie entirely in the magnetic field and thus have the same value of $d\phi_B\, B/dt$. This is true even though the electric field vectors at points along these paths are different, as indicated by the patterns of electric field lines in the figure. For path 3 the induced emf is smaller because the enclosed flux ϕ_B (hence $d\phi_B/dt$) is smaller, and for path 4 the induced emf is zero even though the electric field is not zero at any point on the path.



Motional emf

By Lenz's law, whether you move the magnet toward or away from the loop, a magnetic force resists the motion, requiring your applied force to do positive work. At the same time, thermal energy is produced in the material of the loop because of the material's electrical resistance to the current that is induced by the motion. The energy you transfer to the closed *loop + magnet* system via your applied force ends up in this thermal energy. (For now, we neglect energy that is radiated away from the loop as electromagnetic waves during the induction.) The faster you move the magnet, the more rapidly your applied force does work and the greater the rate at which your energy is transferred to thermal energy in the loop; that is, the power of the transfer is greater. Regardless of how current is induced in a loop, energy is always transferred to thermal energy during the process because of the electrical resistance of the loop (unless the loop is superconducting). For example, when switch S is closed and a current is briefly induced in the left-hand loop, energy is transferred from the battery to thermal energy in that loop.

Figure shows another situation involving induced current. A rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in Fig. show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity **v**.



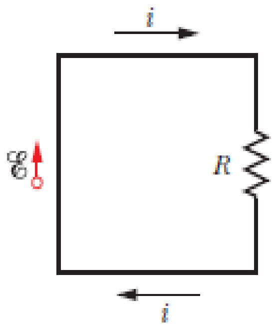
Flux Change. A magnetic field and a conducting loop are in relative motion; in each case the flux of the field through the loop is changing with time. The flux is changing because is changing and in Fig. the flux is changing because the area of the loop still in the magnetic field B is changing, but that difference is not important. The important difference between the two arrangements is that the arrangement of Fig. makes calculations easier. Let us now calculate the rate at which you do mechanical work as you pull steadily on the loop in Fig. As you move the loop to the right in Fig., the portion of its area within the magnetic field decreases. Thus, the flux through the loop also decreases and, according to Faraday's law, a current is produced in the loop. It is the presence of this current that causes the force that opposes your pull.

Induced emf. To find the current, we first apply Faraday's law. When x is the length of the loop still in the magnetic field, the area of the loop still in the field is Lx. Then the magnitude of the flux through the loop is

$$\Phi_B = BA = BLx.$$

As x decreases, the flux decreases. Faraday's law tells us that with this flux decrease, an emf is induced in the loop. We can write the magnitude of this emf as

$$\mathscr{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL\frac{dx}{dt} = BLv,$$



in which we have replaced *dx*/*dt* with *v*, the speed at which the loop moves.

Figure shows the loop as a circuit: induced emf ξ is represented on the left, and the collective resistance *R* of the loop is represented on the right. The direction of the induced current *i* is obtained with a right-hand rule as for decreasing flux; applying the rule tells us that the current must be clockwise, and ξ must have the same direction.

Induced Current. To find the magnitude of the induced current, we cannot apply the loop rule for potential differences in a circuit because we cannot define a potential difference for an induced emf. However, we can apply the equation *i* = ξ/*R*. , this becomes

$$i = \frac{BLv}{R}.$$