A Point Charge in an Electric Field

The electrostatic force F acting on a charged particle located in an external electric field $\bar{E}$ has the direction $\bar{E}$ of if the charge q of the particle is positive and has the opposite direction if q is negative.

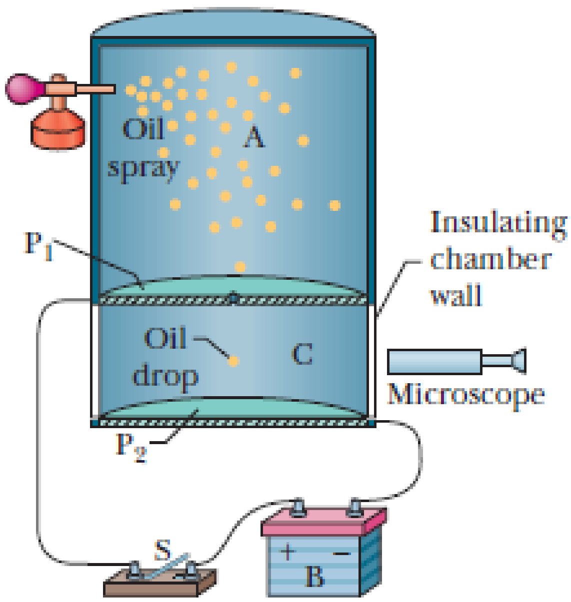
$$\vec{F} = q\vec{E},$$

Measuring the Elementary Charge

Equation $F = q\bar{E}$ played a role in the measurement of the elementary charge e by American physicist Robert A. Millikan in 1910–1913. Figure below is a representation of his apparatus. When tiny oil drops are sprayed into chamber A, some of them become charged, either positively or negatively, in the process. Consider a drop that drifts downward through the small hole in plate P_1 and into chamber C. Let us assume that this drop has a negative charge q.

If switch S in Fig. is open as shown, battery B has no electrical effect on chamber C. If the switch is closed (the connection between chamber C and the positive terminal of the battery is then complete), the battery causes an excess positive charge on conducting plate P_1 and an excess negative charge on conducting plate P_2. The charged plates set up a downward-directed electric field in chamber C. According to Eq. $F = q\bar{E}$ this field exerts an electrostatic force on any charged drop that happens to be in the chamber and affects its motion. In particular,

our negatively charged drop will tend to drift upward. By timing the motion of oil drops with the switch opened and with it closed and thus determining the effect of the charge q, Millikan discovered that the



values of q were always given by

$$q = ne, \text{ for } n = 0, \pm1, \pm2, \pm3, \ldots,$$

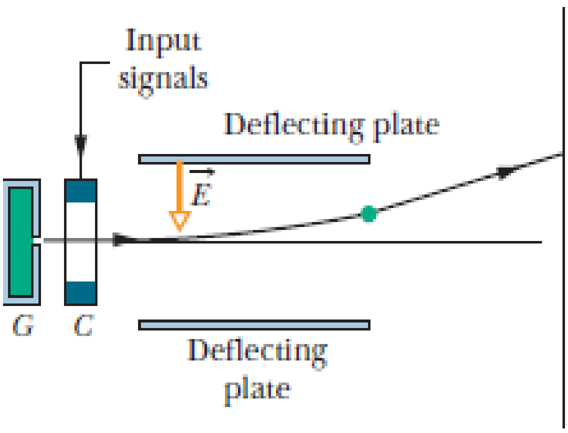
in which e turned out to be the fundamental constant we call the *elementary charge*, 1.60×10^{-19} C. Millikan's experiment is convincing proof that charge is quantized, and he earned the 1923 Nobel Prize in physics in part for this work. Modern measurements of the elementary charge rely on a variety of interlocking experiments, all more precise than the pioneering experiment of Millikan.

Ink-Jet Printing

The need for high-quality, high-speed printing has caused a search for an alternative to impact printing, such as occurs in a standard typewriter. Building up letters by squirting tiny drops of ink at the paper is one such alternative.

Figure below shows a negatively charged drop moving between two conducting deflecting plates, between which a uniform, downward-directed electric field has been set up. The drop is deflected upward according to Eq. $F = q\bar{E}$ and then strikes the paper at a position that is determined by the magnitudes of and the charge q of the drop.

In practice, $\bar{E}$ is held constant and the position of the drop is determined by the charge q delivered to the drop in the charging unit, through which the drop must pass before entering the deflecting system. The charging unit, in turn, is activated by electronic signals that encode the material to be printed.



A Dipole in an Electric Field

We have defined the electric dipole moment of an electric dipole to be a vector that points from the negative to the positive end of the dipole. As you will see, the behaviour of a dipole in a uniform external electric field $\bar{E}$ can be described completely in terms of the two vectors and $\bar{E}$ and p, with no need of any details about the dipole's structure.

To examine this behavior, we now consider such an abstract dipole in a uniform external electric field Ē, as shown in Fig *a*. We assume that the dipole is a rigid structure that consists of two centers of opposite charge, each of magnitude *q*, separated by a distance *d*. The dipole moment makes an angle θ with field Ē.

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig. *a*) and with the same magnitude $F = qE$. Thus, *because the field is uniform,* the net force on the dipole from the field is zero and the centre of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque τ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance *x* from one end and thus a distance $d - x$ from the other end. As $\tau = rF \sin \theta$, we can write the magnitude of the net torque t as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta.$$

We can also write the magnitude of t: in terms of the magnitudes of the electric field *E* and the dipole moment $p = qd$. To do so, we substitute qE for *F* and p/q for *d* in above eq., finding that the magnitude of τ is

$$\tau = pE \sin \theta.$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

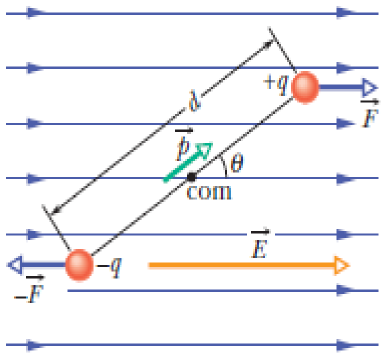
Vectors *p* and Ē are shown in Fig. *b*. The torque acting on a dipole tends to rotate *p*: (hence the dipole) into the direction of field Ē, thereby reducing θ. In Fig., such rotation is clockwise. We can represent a torque τ that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque τ. With that notation, the torque of Fig. is

$$\tau = -pE \sin \theta.$$

Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment *p* is lined up with the field Ē (then $\tau = p \times \bar{E} = 0$. It has greater potential energy in all other orientations. Thus the dipole is like a pendulum, which has *its* least gravitational potential energy in *its* equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

In any situation involving potential energy, we are free to define the zero potential-energy configuration in an arbitrary way because only differences in potential energy have physical meaning. The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ in Fig. is 90°. We then can find the potential energy U of the dipole at any other value of θ with ($\Delta U = -W$) by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.With the aid of Eq. ($W = \int \tau d\theta$) and above eq., we find that the potential energy U at any angle θ is



(a)

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta.$$

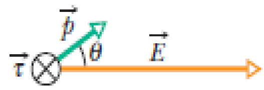
Evaluating the integral leads to

$$U = -pE\cos\theta.$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$

The dipole is being torqued into alignment.



(b)

Microwave Cooking

Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field Ē within the oven and thus also within the food. From eq.

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

we see that any electric field Ē produces a torque on an electric dipole moment p to align with p. Because the oven's Ē oscillates, the water molecules continuously flip-flop in a frustrated attempt to align with Ē.

Energy is transferred from the electric field to the thermal energy of the water (and thus of the food) where three water molecules happened to have bonded together to form a group. The flip-flop breaks some of the bonds. When the molecules reform the bonds, energy is transferred to the random motion of the group and then to the surrounding molecules. Soon, the thermal energy of the water is enough to cook the food.