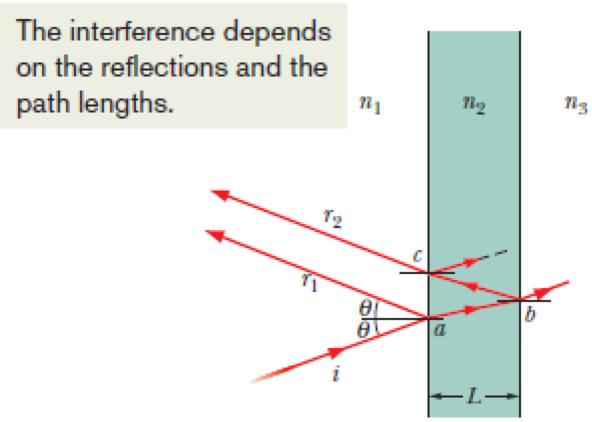
Interference from Thin Films

The colors on a sunlit soap bubble or an oil slick are caused by the interference of light waves reflected from the front and back surfaces of a thin transparent film. The thickness of the soap or oil film is typically of the order of magnitude of the wavelength of the (visible) light involved. (Greater thicknesses spoil the coherence of the light needed to produce the colors due to interference.)

Figure shows a thin transparent film of uniform thickness L and index of refraction n_2, illuminated by bright light of wavelength l from a distant point source. For now, we assume that air lies on both sides of the film and thus that $n_1 = n_3$ in Fig. For simplicity, we also assume that the light rays are almost perpendicular to the film ($\theta = 0$).We are interested in whether the film is bright or dark to an observer viewing it almost perpendicularly. (Since the film is brightly illuminated, how could it possibly be dark? You will see.) The incident light, represented by ray *i*, intercepts the front (left) surface of the film at point *a* and undergoes both reflection and refraction there. The reflected ray r_1 is intercepted by the observer's eye. The refracted light crosses the film to point *b* on the back surface, where it undergoes both reflection and refraction. The light reflected at *b* crosses back through the film to point *c*, where it undergoes both reflection and refraction. The light refracted at *c*, represented by ray r_2, is intercepted by the observer's eye.



If the light waves of rays r_1 and r_2 are exactly in phase at the eye, they produce an interference maximum and region *ac* on the film is bright to the observer. If they are exactly out of phase, they produce an interference minimum and region *ac* is dark to the observer, *even though it is illuminated.* If there is some intermediate phase difference, there are intermediate interference and brightness.

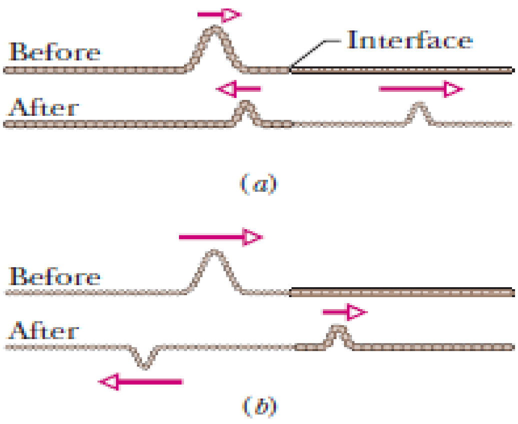
The Key. Thus, the key to what the observer sees is the phase difference between the waves of rays r_1 and r_2. Both rays are derived from the same ray *i*, but the path involved in producing r_2 involves light traveling twice across the film (*a* to *b*, and then *b* to *c*), whereas the path involved in producing r_1 involves no travel through the film. Because u is about zero, we approximate the path length difference between the waves of r_1 and r_2 as $2L$. However, to find the phase difference between the waves, we cannot just find the number of wavelengths l that is equivalent to a path length difference of $2L$. This simple approach is impossible for two reasons: (1) the path length difference occurs in a medium other than air, and (2) reflections are involved, which can change the phase.

The phase difference between two waves can change if one or both are reflected.

Let's next discuss changes in phase that are caused by reflections.

Reflection Phase Shifts

Refraction at an interface never causes a phase change—but reflection can, depending on the indexes of refraction on the two sides of the interface. Figure below shows what happens when reflection causes a phase change, using as an example pulses on a denser string (along which pulse travel is relatively slow) and a lighter string (along which pulse travel is relatively fast). When a pulse traveling relatively slowly along the denser string in Fig. *a* reaches the interface with the lighter string, the pulse is partially transmitted and partially reflected, with no change in orientation. For light, this situation corresponds to the incident wave traveling in the medium of greater index of refraction *n* (recall that greater *n* means slower speed). In that case, the wave that is reflected at the interface does not undergo a change in phase; that is, its *reflection phase shift* is zero.



When a pulse traveling more quickly along the lighter string in Fig. *b* reaches the interface with the denser string, the pulse is again partially transmitted and partially reflected. The transmitted pulse again has the same orientation as the incident pulse, but now the reflected pulse is inverted. For a sinusoidal wave, such an inversion involves a phase change of p rad, or half a wavelength. For light, this situation corresponds to the incident wave traveling in the medium of lesser index of refraction (with greater speed). In that case, the wave that is reflected at the interface undergoes a phase shift of p rad, or half a wavelength. We can summarize these results for light in terms of the index of refraction of the medium off which (or from which) the light reflects:

Reflection	Reflection phase shift
Off lower index	0
Off higher index	0.5 wavelength

This might be remembered as "higher means half."

Equations for Thin-Film Interference

In this chapter we have now seen three ways in which the phase difference between two waves can change:

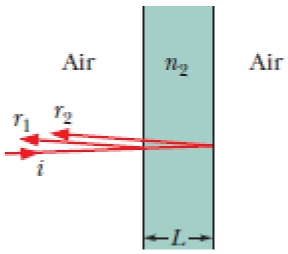
1. by reflection

2. by the waves traveling along paths of different lengths

3. by the waves traveling through media of different indexes of refraction

When light reflects from a thin film, producing the waves of rays r_1 and r_2, all three ways are involved. Let us consider them one by one.

Reflection Shift. We first re-examine the two reflections. At point *a* on the front interface, the incident wave (in air) reflects from the medium having the higher of the two indexes of refraction; so the wave of reflected ray r_1 has its phase shifted by 0.5 wavelength. At point *b* on the back interface, the incident wave reflects from the medium (air) having the lower of the two indexes of refraction; so the wave reflected there is not shifted in phase by the reflection, and thus neither is the portion of it that exits the film as ray r_2. which refers to the simplified drawing in Fig. below for a thin film in air. So far, as a result of the reflection phase shifts, the waves of r_1 and r_2 have a phase difference of 0.5 wavelength and thus are exactly out of phase.



Path Length Difference. Now we must consider the path length difference $2L$ that occurs because the wave of ray r_2 crosses the film twice. If the waves of r_1 and r_2 are to be exactly in phase so that they produce fully constructive interference, the path length $2L$ must cause an additional phase difference of 0.5, 1.5, 2.5, . . . wavelengths. Only then will the net phase difference be an integer number of wavelengths.

Thus, for a bright film, we must have

$$2L = \frac{\text{odd number}}{2} \times \text{wavelength} \quad \text{(in-phase waves)}.$$

The wavelength we need here is the wavelength λn_2 of the light in the medium containing path length $2L$—that is, in the medium with index of refraction n_2.

Thus, we can rewrite above equation as

$$2L = \frac{\text{odd number}}{2} \times \lambda_{n2} \quad \text{(in-phase waves)}.$$

If, instead, the waves are to be exactly out of phase so that there is fully destructive interference, the path length $2L$ must cause either no additional phase difference or a phase difference of 1, 2, 3, . . . wavelengths. Only then will the net phase difference be an odd number of half-wavelengths. For a dark film, we must have

$$2L = \text{integer} \times \text{wavelength} \quad \text{(out-of-phase waves)}.$$

where, again, the wavelength is the wavelength ln2 in the medium containing $2L$. Thus, this time we have

$$2L = \text{integer} \times \lambda_{n2} \quad \text{(out-of-phase waves)}.$$

Now we can use ($\lambda_n = \lambda/n$) to write the wavelength of the wave of ray r_2 inside the film as

$$\lambda_{n2} = \frac{\lambda}{n_2},$$

where l is the wavelength of the incident light in vacuum (and approximately also in air). Substituting above equation into in-phase waves equation and replacing “odd number/2” with (m+1/2) give us

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air)}.$$

Similarly, with m replacing “integer,” out of phase wave equation yields

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$

For a given film thickness L, above two equations tell us the wavelengths of light for which the film appears bright and dark, respectively, one wavelength for each value of m. Intermediate wavelengths give intermediate brightnesses. For a given wavelength l, above two equations tell us the thicknesses of the films that appear bright and dark in that light, respectively, one thickness for each value of m. Intermediate thicknesses give intermediate brightnesses.

Heads Up. (1) For a thin film surrounded by air, above two equations corresponds to bright reflections and to no reflections, respectively. For transmissions, the roles of the equations are reversed (after all, if the light is brightly reflected, then it is not transmitted, and vice versa). (2) If we have a different set of values of the indexes of refraction, the roles of the equations may be reversed. (3) The index of refraction in the equations is that of the thin film, where the path length difference occurs.

Film Thickness Much Less Than λ

A special situation arises when a film is so thin that L is much less than λ, say, $L < 0.1\lambda$. Then the path length difference $2L$ can be neglected, and the phase difference between r_1 and r_2 is due *only* to reflection phase shifts. If the film of Fig. above, where the reflections cause a phase difference of 0.5 wavelength, has thickness $L < 0.1\lambda$, then r_1 and r_2 are exactly out of phase, and thus the film is dark, regardless of the wavelength and intensity of the light.This special situation corresponds to m _ 0 in above equation. We shall count *any* thickness $L < 0.1\lambda$ as being the least thickness specified by above equation to make the film of Fig. above dark. (Every such thickness will correspond to $m = 0$.) The next greater thickness that will make the film dark is that corresponding to $m = 1$.