Displacement Current

If you compare the two terms on the right side of below equation,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \quad \text{(Ampere–Maxwell law).}$$

you will see that the product $\varepsilon_0(d\phi_E/dt)$ must have the dimension of a current. In fact, that product has been treated as being a fictitious current called the **displacement current** i_d:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(displacement current).}$$

"Displacement" is poorly chosen in that nothing is being displaced, but we are stuck with the word

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}} \quad \text{(Ampere–Maxwell law),}$$

in which $i_{d,\text{enc}}$ is the displacement current that is encircled by the integration loop.
Let us again focus on a charging capacitor with circular plates, as in Fig.*a*. The real current i that is charging the plates changes the electric field between the plates. The fictitious displacement current *id* between the plates is associated with that changing field $\bar{E}$. Let us relate these two currents. The charge q on the plates at any time is related to the magnitude E of the field between the plates at that time and the plate area A:

$$q = \varepsilon_0 A E.$$

To get the real current i, we differentiate above eq., with respect to time, finding

$$\frac{dq}{dt} = i = \varepsilon_0 A \frac{dE}{dt}.$$

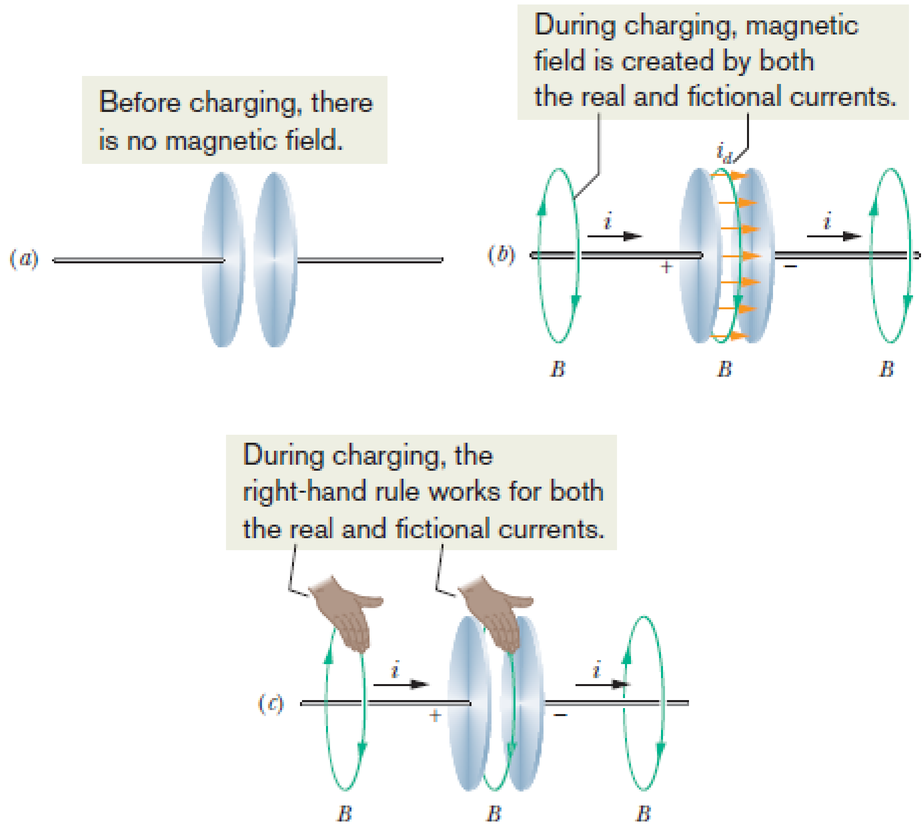
Assuming that the electric field E between the two plates is uniform (we neglect any fringing), we can replace the electric flux ϕ_E in that equation with EA.

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}.$$

Same Value. Comparing above eqs., we see that the real current i charging the capacitor and the fictitious displacement current i_d between the plates have the same value:

$$i_d = i \quad \text{(displacement current in a capacitor).}$$

Thus, we can consider the fictitious displacement current i_d to be simply a continuation of the real current i from one plate, across the capacitor gap, to the other plate. Because the electric field is uniformly spread over the plates, the same is true of this fictitious displacement current i_d, as suggested by the spread of current arrows in Fig. *b*.



Although no charge actually moves across the gap between the plates, the idea of the fictitious current *id* can help us to quickly find the direction and magnitude of an induced magnetic field, as follows.

Finding the Induced Magnetic Field

The direction of the magnetic field produced by a real current *i* is found by using the right-hand rule. We can apply the same rule to find the direction of an induced magnetic field produced by a fictitious displacement current i_d, as is shown in the center of Fig. *c* for a capacitor. We can also use i_d to find the magnitude of the magnetic field induced by a charging capacitor with parallel circular plates of radius *R*. We simply consider the space between the plates to be an imaginary circular wire of radius *R* carrying the imaginary current *id*. The magnitude of the magnetic field at a point inside the capacitor at radius *r* from the center is

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r \quad \text{(inside a circular capacitor).}$$

The magnitude of the magnetic field at a point outside the capacitor at radius *r* is

$$B = \frac{\mu_0 i_d}{2\pi r} \quad \text{(outside a circular capacitor).}$$