Electric Potential

→ "The amount of work done in bringing a unit positive charge from infinity to a point P is called Electric Potential."

or

→ "The electric potential energy per unit positive test charge is called Electric Potential."

→ Electric potential is denoted by **"V"**.

→ *Formula:*

$$V = \frac{W}{qo} = \frac{F.d}{qo} = \frac{qoE.d}{qo} = E.d \qquad (\because F=q_oE)$$

$$\Delta V = \frac{\Delta U}{qo}$$

→ It is a scalar quantity can be both positive or negative.

→ *Unit:* Its unit is **"volt"**.

$$\text{volt} = \frac{joule}{coulomb} = \frac{J}{C}$$

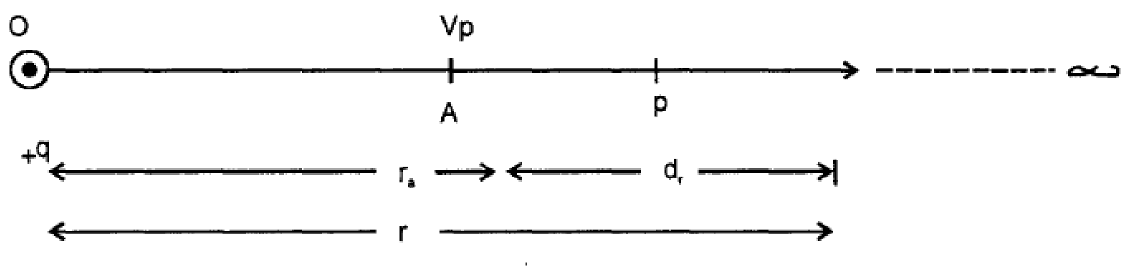
→ The change in Δ**U** is related to work **W** in electric field as

$$W = -\Delta U$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Electric Potential Due To A Point Charge

Consider the electric field due to a point charge +q placed at point 0, Suppose that V_A is electric potential at point A. Whose distance from the charge +q is r_A



Suppose that any instant, the test charge at point P. The electric field E due to charge +q exerts force $q_o\bar{E}$ on test charge placed at point P. The test charge q_o can be moved against this force without acceleration by applying on external force is given by.

$$\mathbf{F} = - q_0\bar{E}$$

If the test charge qo is moved through small displacement dr = AP, then small work done is given by.

$$dw = F.\, dr \qquad [\because\ W = F \times D\ ]$$

$$dw = (- q_0\vec{E}\ )\ dr$$

Now electric field intensity E at a point is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Putting the value of $\bar{E}$, we have,

$$dw = - q_0\ (\frac{1}{4\pi\epsilon_0}\frac{q}{r^2})\ dr$$

$$dw = \frac{-1}{4\pi\epsilon_0}\frac{q\, q_0}{r^2}\ dr \text{ --}$$

Now we determine the total work done in moving a unit positive test charge from infinity to point P at distance r from charge +q at point O,

$$\text{Then,}\quad W = \int_\infty^r dw = \int_\infty^r \frac{1}{4\pi\epsilon_0}\frac{q\, q_0}{r^2}\ dr$$

$$W = -\frac{1}{4\pi\epsilon_0}\, q\, q_0 \int_\infty^r \frac{1}{r^2}\ dr$$

$$= -\frac{1}{4\pi\epsilon_0}\, qq_0\ [\frac{r^{-1}}{-1}]_\infty^r$$

$$= \frac{1}{4\pi\epsilon_0}\, q\, q_0\ [\frac{1}{r}]_\infty^r$$

$$= \frac{1}{4\pi\epsilon_0}\, q\, q_0\ \left[\frac{1}{r}\right]_\infty^r$$

$$= \frac{1}{4\pi\epsilon_0}\, qq_0 \left[\frac{1}{r} - 0\right]$$

$$W = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

Now potential at a point A is given by

$$V_A = \frac{W}{q_0}$$

$$V_A = \frac{\frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}}{q_0}$$

$$\boxed{V_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

Electric Potential Due To Dipole

Now let us find the potential at an arbitrary point P in Fig. *a*. At P, the positively charged particle (at distance $r_{(+)}$) sets up potential $V_{(+)}$ and the negatively charged particle (at distance $r_{(-)}$) sets up potential $V_{(-)}$.Then the net potential at P is given by as

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

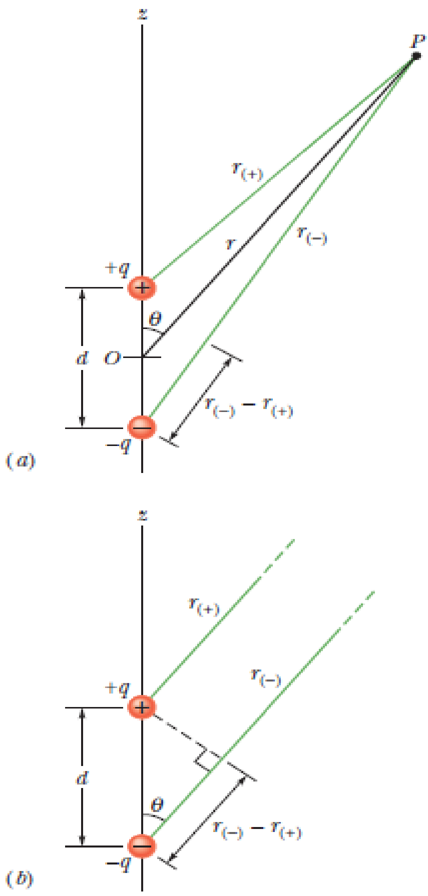
$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that r>>d, where d is the distance between the charges and r is the distance from the dipole's midpoint to P.

In that case, we can approximate the two lines to P as being parallel and their length difference as being the leg of a right triangle with hypotenuse d (Fig. *b*). Also, that difference is so small that the product of the lengths is approximately r^2, *thus*

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

If we substitute these quantities into above equations, we can approximate V to be



$$V = \frac{q}{4\pi\varepsilon_0} \frac{d \cos \theta}{r^2},$$

where θ is measured from the dipole axis as shown in Fig. *a*. We can now V as

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p \cos \theta}{r^2} \quad \text{(electric dipole)},$$