Double Slit Interference

Coherence

For the interference pattern to appear on viewing screen, the light waves reaching any point on the screen must have a phase difference that does not vary in time. The waves passing through slits are portions of the single light wave that illuminates the slits. Because the phase difference remains constant, the light from slits is said to be completely **coherent.**

Sunlight and Fingernails. Direct sunlight is partially coherent; that is, sunlight waves intercepted at two points have a constant phase difference only if the points are very close. If you look closely at your fingernail in bright sunlight, you can see a faint interference pattern called *speckle* that causes the nail to appear to be covered with specks. You see this effect because light waves scattering from very close points on the nail are sufficiently coherent to interfere with one another at your eye. The slits in a double-slit experiment, however, are not close enough, and in direct sunlight, the light at the slits would be **incoherent.** To get coherent light, we would have to send the sunlight through a single slit because that single slit is small, light that passes through it is coherent. In addition, the smallness of the slit causes the coherent light to spread via diffraction to illuminate both slits in the double-slit experiment.

Incoherent Sources. If we replace the double slits with two similar but independent monochromatic light sources, such as two fine incandescent wires, the phase difference between the waves emitted by the sources varies rapidly and randomly. (This occurs because the light is emitted by vast numbers of atoms in the wires, acting randomly and independently for extremely short times—of the order of nanoseconds.) As a result, at any given point on the viewing screen, the interference between the waves from the two sources varies rapidly and randomly between fully constructive and fully destructive. The eye (and most common optical detectors) cannot follow such changes, and no interference pattern can be seen. The fringes disappear, and the screen is seen as being uniformly illuminated.

Coherent Source. A *laser* differs from common light sources in that its atoms emit light in a cooperative manner, thereby making the light coherent. Moreover, the light is almost monochromatic, is emitted in a thin beam with little spreading, and can be focused to a width that almost matches the wavelength of the light.

Intensity in Double-Slit Interference

Equations below

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright fringes).}$$

And

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark fringes)}.$$

tell us how to locate the maxima and minima of the double-slit interference pattern on screen *C* as a function of the angle θ in that figure. Here we wish to derive an expression for the intensity *I* of the fringes as a function of θ. The light leaving the slits is in phase. However, let us assume that the light waves from the two slits are not in phase when they arrive at point *P*. Instead, the electric field components of those waves at point *P* are not in phase and vary with time as

$$E_1 = E_0 \sin \omega t$$
$$E_2 = E_0 \sin(\omega t + \phi),$$

where v is the angular frequency of the waves and f is the phase constant of wave E_2. Note that the two waves have the same amplitude E_0 and a phase difference of ϕ. Because that phase difference does not vary, the waves are coherent. We shall show that these two waves will combine at *P* to produce an intensity *I* given by

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

and that

$$\phi = \frac{2\pi d}{\lambda} \sin\theta.$$

In above eq., I_0 is the intensity of the light that arrives on the screen from one slit when the other slit is temporarily covered. We assume that the slits are so narrow in comparison to the wavelength that this single-slit intensity is essentially uniform over the region of the screen in which we wish to examine the fringes.

Above equations, which together tell us how the intensity *I* of the fringe pattern varies with the angle θ, necessarily contain information about the location of the maxima and minima. Let us see if we can extract that information to find equations about those locations.

Maxima. intensity maxima will occur when

$$\tfrac{1}{2}\phi = m\pi, \qquad \text{for } m = 0, 1, 2, \ldots .$$

$$2m\pi = \frac{2\pi d}{\lambda} \sin\theta, \qquad \text{for } m = 0, 1, 2, \ldots$$

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima)},$$

which is exactly the expression that we derived earlier for the locations of the maxima.

Minima. The minima in the fringe pattern occur when

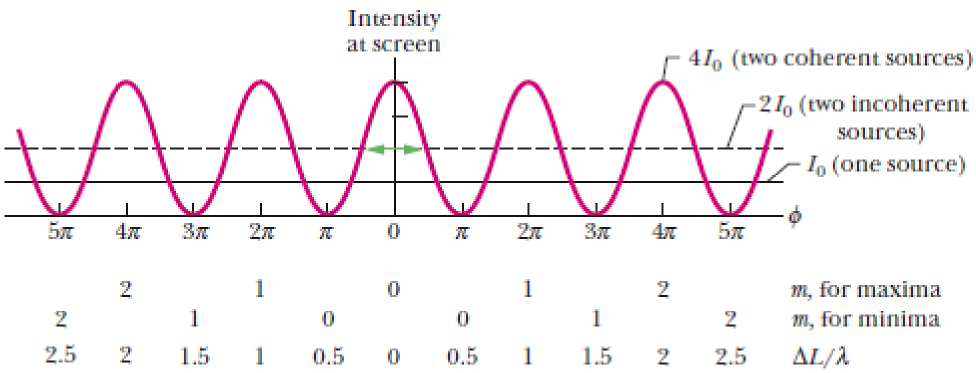
$$\tfrac{1}{2}\phi = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 0, 1, 2, \ldots.$$

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima)},$$

which is just the expression we derived earlier for the locations of the fringe minima.

Figure below, which is a plot of Eq. $I = 4I_0\cos^2(1/2\phi)$, shows the intensity of double-slit interference patterns as a function of the phase difference f between the waves at the screen. The horizontal solid line is I_0, the (uniform) intensity on the screen when one of the slits is covered up. Note in Eq. $I = 4I_0\cos^2(1/2\phi)$ and the graph that the intensity I varies from zero at the fringe minima to $4I_0$ at the fringe maxima. If the waves from the two sources (slits) were *incoherent*, so that no enduring phase relation existed between them, there would be no fringe pattern and the intensity would have the uniform value $2I_0$ for all points on the screen; the horizontal dashed line in Fig. below shows this uniform value.

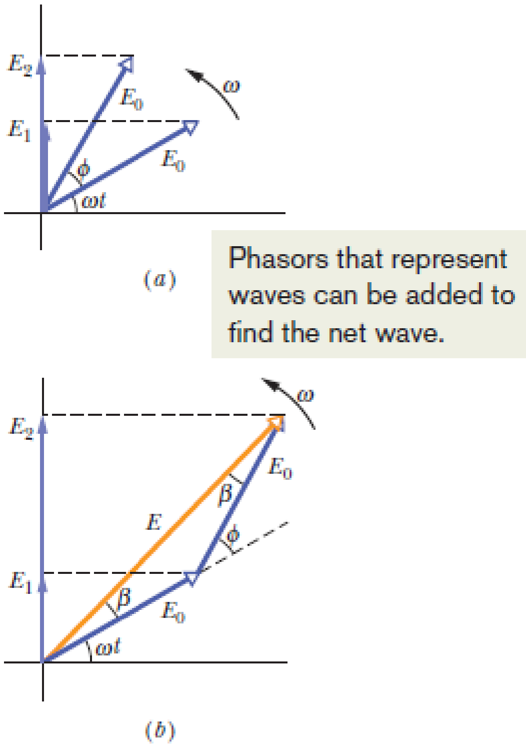
Interference cannot create or destroy energy but merely redistributes it over the screen. Thus, the *average* intensity on the screen must be the same $2I_0$ regardless of whether the sources are coherent. This follows at once from Eq. $I = 4I_0\cos^2(1/2\phi)$; if we substitute , the average value of the cosine-squared function, this equation reduces to I_{avg} $2I_0$.



We shall combine the electric field components E_1 and E_2, given by Eqs. $E_1 = E_0\sin\omega t$ and $E_2 = E_0\sin(\omega t+\phi)$, respectively.

In Fig. *a*, the waves with components E_1 and E_2 are represented by phasors of magnitude E_0 that rotate around the origin at angular speed v. The values of E_1 and E_2 at any time are the projections of the corresponding phasors on the vertical axis. Figure *a* shows the phasors and their projections at an arbitrary time *t*. Consistent with Eqs. $E_1 = E_0\sin\omega t$ and $E_2 = E_0\sin(\omega t+\phi)$, the phasor for E_1 has a rotation angle ωt and the phasor for E_2 has a rotation angle $\omega t + \phi$ (it is phase-shifted ahead of E_1). As each phasor rotates, its projection on the vertical axis varies with time in the same way that the sinusoidal functions of Eqs. $E_1 = E_0 \sin\omega t$ and $E_2 = E_0 \sin(\omega t+\phi)$ vary with time.

To combine the field components E_1 and E_2 at any point P, we add their phasors vectorially, as shown in fig. *b*.



The magnitude of the vector sum is the amplitude E of the resultant wave at point P, and that wave has a certain phase constant β. To find the amplitude E in Fig. *b*, we first note that the two angles marked β are equal because they are opposite equal-length sides of a triangle. From the theorem (for triangles) that an exterior angle (here ϕ, as shown in Fig. *b*) is equal to the sum of the two opposite interior angles (here that sum is $\beta + \beta$), we see that $\beta = 1/2\phi$. Thus, we have

$$\begin{aligned} E &= 2(E_0 \cos\beta) \\ &= 2E_0 \cos\tfrac{1}{2}\phi. \end{aligned}$$

If we square each side of this relation, we obtain

$$E^2 = 4E_0^2 \cos^2 \tfrac{1}{2}\phi.$$

Intensity. we know that the intensity of an electromagnetic wave is proportional to the square of its amplitude. Therefore, the waves we are combining in Fig. *b*, whose amplitudes are E_0, each has an intensity I_0 that is proportional to , and the resultant wave, with amplitude E, has an intensity I that is proportional to E_2.Thus,

$$\frac{I}{I_0} = \frac{E^2}{E_0^2}.$$

Substituting $E^2 = 4E_0^2\cos^2(1/2\phi)$ into this equation and rearranging then yield which is $I = 4I_0\cos^2(1/2\phi)$,

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

which we set out to prove.

This suggests

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \frac{2\pi}{\lambda}\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}.$$

So above equation for the phase difference between the two waves arriving at point *P* on the screen becomes

$$\phi = \frac{2\pi d}{\lambda} \sin\theta,$$

Combining More Than Two Waves

In a more general case, we might want to find the resultant of more than two sinusoidally varying waves at a point. Whatever the number of waves is, our general procedure is this:

1. Construct a series of phasors representing the waves to be combined. Draw them end to end, maintaining the proper phase relations between adjacent phasors.

2. Construct the vector sum of this array. The length of this vector sum gives the amplitude of the resultant phasor. The angle between the vector sum and the first phasor is the phase of the resultant with respect to this first phasor. The projection of this vector-sum phasor on the vertical axis gives the time variation of the resultant wave.