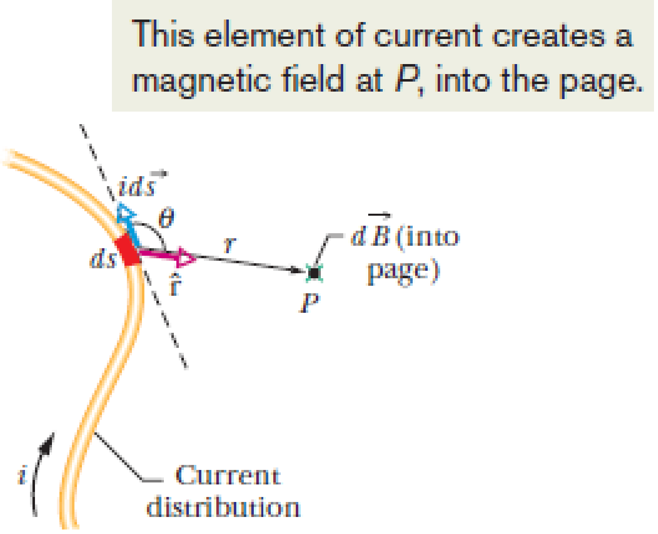
Calculating the Magnetic Field Due to a Current

Figure shows a wire of arbitrary shape carrying a current *i*. We want to find the magnetic field at a nearby point *P*. We first mentally divide the wire into differential elements *ds* and then define for each element a length vector ds that has length *ds* and whose direction is the direction of the current in *ds*.



We can then define a differential *current-length element* to be *ids;* we wish to calculate the field dB produced at *P* by a typical current-length element. From experiment we find that magnetic fields, like electric fields, can be superimposed to find a net field. Thus, we can calculate the net field B at *P* by summing, via integration, the contributions dB from all the current-length elements. However, this summation is more challenging than the process associated with electric fields because of a complexity; whereas a charge element *dq* producing an electric field is a scalar, a current-length element *ids* producing a magnetic field is a vector, being the product of a scalar and a vector.

Magnitude. The magnitude of the field dB produced at point *P* at distance *r* by a current length element *i* turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

where u is the angle between the directions of and ds and **r**, a unit vector that points from *ds* toward *P*. Symbol u_0 is a constant, called the *permeability constant*, whose value is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\,\mathrm{T \cdot m/A}.$$

Direction. The direction of , shown as being into the page in figure, is that of the cross product ds×r. We can therefore write above equation in vector form as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathbf{r}}}{r^2} \quad \text{(Biot–Savart law).}$$

This vector equation and its scalar form, are known as the **law of Biot and Savart** (rhymes with "Leo and bazaar"). The law, which is experimentally deduced, is an inverse-square law. We shall use this law to calculate the net magnetic field B produced at a point by various distributions of current. Here is one easy distribution: If current in a wire is either directly toward or directly away from a point *P* of measurement, can you see from equation that the magnetic field at *P* from the current is simply zero (the angle θ is either 0 for *toward* or 180° for *away*, and both result in sin θ = 0)?

Magnetic Field Due to a Current in a Long Straight Wire

Shortly we shall use the law of Biot and Savart to prove that the magnitude of the magnetic field at a perpendicular distance *R* from a long (infinite) straight wire carrying a current *i* is given by

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{(long straight wire).}$$

Directions. Plugging values into above equation to find the field magnitude *B* at a given radius is easy. What is difficult for many students is finding the direction of a field vector B at a given point. The field lines form circles around a long straight wire, and the field vector at any point on a circle must be tangent to the circle. That means it must be perpendicular to a radial line extending to the point from the wire. But there are two possible directions for that perpendicular vector. One is correct for current into the figure, and the other is correct for current out of the figure. How can you tell which is which? Here is a simple right-hand rule for telling which vector is correct:

Curled–straight right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.