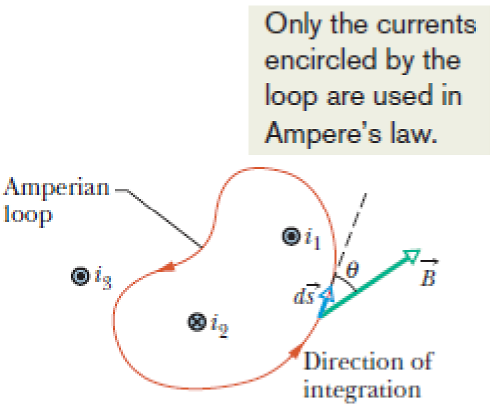
## Ampere’s Law

We can find the net electric field due to *any* distribution of charges by first writing the differential electric field $d\bar{E}$ due to a charge element and then summing the contributions of $d\bar{E}$ from all the elements. However, if the distribution is complicated, we may have to use a computer. Recall, however, that if the distribution has planar, cylindrical, or spherical symmetry, we can apply Gauss’ law to find the net electric field with considerably less effort. Similarly, we can find the net magnetic field due sto *any* distribution of currents by first writing the differential magnetic field dB due to a current-length element and then summing the contributions of from all the elements. Again we may have to use a computer for a complicated distribution. However, if the distribution has some symmetry, we may be able to apply **Ampere’s law** to find the magnetic field with considerably less effort. This law, which can be derived from the Biot–Savart law, has traditionally been credited to André-Marie Ampère (1775–1836), for whom the SI unit of current is named. However, the law actually was advanced by English physicist James Clerk Maxwell. Ampere’s law is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).}$$

The loop on the integral sign means that the scalar (dot) product B.ds is to be integrated around a *closed* loop, called an *Amperian loop.* The current $i_{enc}$ is the *net* current encircled by that closed loop. To see the meaning of the scalar product B.ds and its integral, let us first apply Ampere’s law to the general situation of Fig. The figure shows cross sections of three long straight wires that carry currents $i_1$, $i_2$, and $i_3$ either directly into or directly out of the page. An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third. The counter clockwise direction marked on the loop indicates the arbitrarily chosen direction of integration of above equation.




To apply Ampere’s law, we mentally divide the loop into differential vector elements that are everywhere directed along the tangent to the loop in the direction of integration.

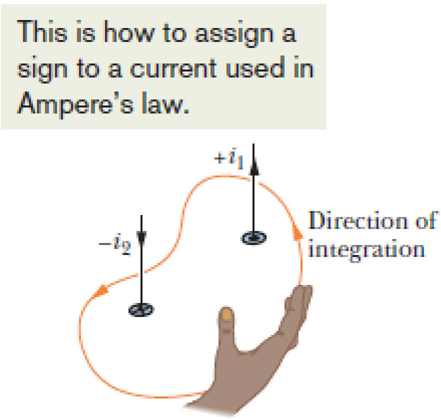
Assume that at the location of the element in figure, the net magnetic field due to the three currents is B. Because the wires are perpendicular to the page, we know that the magnetic field at ds due to each current is in the plane of figure; thus, their net magnetic field B at ds must also be in that plane. However, we do not know the orientation of B within the plane. B is arbitrarily drawn at an angle θ to the direction of B. The scalar product B.ds on the left side of above equation is equal to $B \cos \theta \, ds$. Thus, Ampere's law can be written as

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \theta \, ds = \mu_0 i_{\text{enc}}.$$

We can now interpret the scalar product as being the product of a length *ds* of the Amperian loop and the field component *B* cosθ ds tangent to the loop. Then we can interpret the integration as being the summation of all such products around the entire loop.

***Signs.*** When we can actually perform this integration, we do not need to know the direction of B before integrating. Instead, we arbitrarily assume to be generally in the direction of integration. Then we use the following curled–straight right-hand rule to assign a plus sign or a minus sign to each of the currents that make up the net encircled current $i_{\text{enc}}$:

***Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.***



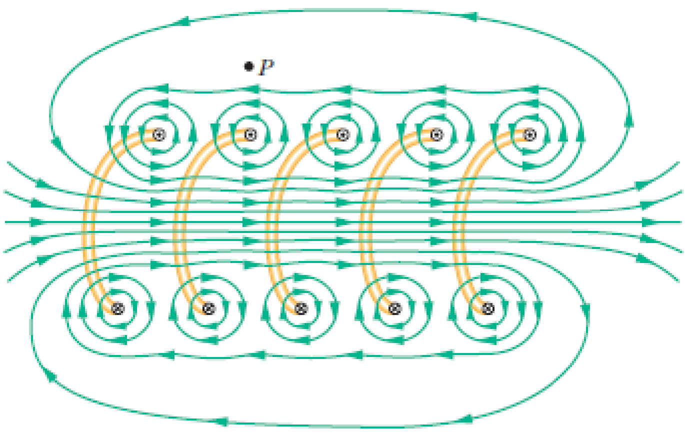

Finally, we solve above equation for the magnitude of B. If *B* turns out positive, then the direction we assumed for is correct. If it turns out negative, we neglect the minus sign and redraw in the opposite direction.

## Solenoids

### Magnetc field of a solenoid

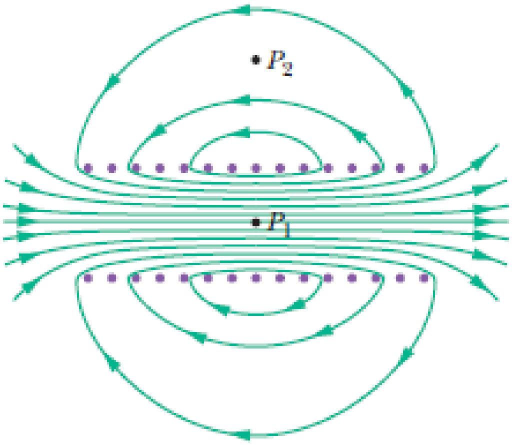
We now turn our attention to another situation in which Ampere's law proves useful. It concerns the magnetic field produced by the current in a long, tightly wound helical coil of wire. Such a coil is called a **solenoid**. We assume that the length of the solenoid is much greater than the diameter.

Figure below shows a section through a portion of a "stretched-out" solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (*windings*) that make up the solenoid. For points very close to a turn, the wire behaves magnetically almost like a long straight wire, and the lines of B there are almost concentric circles. Figure suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire, B is approximately parallel to the (central) solenoid axis. In the limiting case of an *ideal solenoid*, which is infinitely long and consists of tightly packed (*close-packed*) turns of square wire, the field inside the coil is uniform and parallel to the solenoid axis.




At points above the solenoid, such as *P* in above figure, the magnetic field set up by the upper parts of the solenoid turns (these upper turns are marked (.)) is directed to the left (as drawn near *P*) and tends to cancel the field set up at *P* by the lower parts of the turns (these lower turns are marked (×)), which is directed to the right (not drawn). In the limiting case of an ideal solenoid, the magnetic field outside the solenoid is zero. Taking the external field to be zero is an excellent assumption for a real solenoid if its length is much greater than its diameter and if we consider external points such as point *P* that are not at either end of the solenoid. The direction of the magnetic field along the solenoid axis is given by a curled – straight right-hand rule: Grasp the solenoid with your right hand so that your fingers follow the direction of the current in the windings; your extended right thumb then points in the direction of the axial magnetic field.

Figure 29-19 shows the lines of B for a real solenoid. The spacing of these lines in the central region shows that the field inside the coil is fairly strong and uniform over the cross section of the coil. The external field, however, is relatively weak.

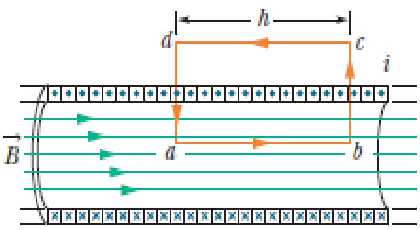



***Ampere's Law.*** Let us now apply Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

to the ideal solenoid of figure, where is uniform within the solenoid and zero outside it, using the rectangular Amperian loop *abcda*. We write as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$




The first integral on the right of above equation is *Bh*, where *B* is the magnitude of the uniform field inside the solenoid and *h* is the (arbitrary) length of the segment from *a* to *b*. The second and fourth integrals are zero because for every element *ds* of these segments, B either is perpendicular to *ds* or is zero, and thus the product is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because $B = 0$ at all external points. Thus, for the entire rectangular loop has the value *Bh*.

***Net Current.*** The net current $i_{\text{enc}}$ encircled by the rectangular Amperian loop in figure is not the same as the current *i* in the solenoid windings because the windings pass more than once through this loop. Let *n* be the number of turns per unit length of the solenoid; then the loop encloses *nh* turns and

$$i_{enc} = i(nh).$$

Ampere's law then gives us

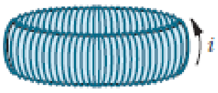
$$Bh = \mu_0 inh$$

or $$B = \mu_0 in \quad \text{(ideal solenoid).}$$

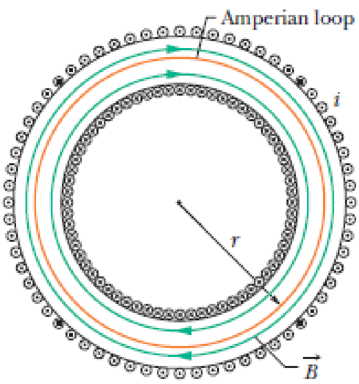
Although we derived above equation for an infinitely long ideal solenoid, it holds quite well for actual solenoids if we apply it only at interior points and well away from the solenoid ends. Equation is consistent with the experimental fact that the magnetic field magnitude *B* within a solenoid does not depend on the diameter or the length of the solenoid and that *B* is uniform over the solenoidal cross section. A solenoid thus provides a practical way to set up a known uniform magnetic field for experimentation, just as a parallel-plate capacitor provides a practical way to set up a known uniform electric field

## Magnetic Field of a Toroid

Figure *a* shows a **toroid,** which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet. What magnetic field B is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet. From the symmetry, we see that the lines of B form concentric circles



(*a*)



(*b*)

inside the toroid, directed as shown in Fig. *b*. Let us choose a concentric circle of radius *r* as an Amperian loop and traverse it in the clockwise direction. Ampere's law yields

$$(B)(2\pi r) = \mu_0 iN,$$

where $i$ is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and $N$ is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \quad \text{(toroid).}$$

In contrast to the situation for a solenoid, $B$ is not constant over the cross section of a toroid.

It is easy to show, with Ampere's law, that $B = 0$ for points outside an ideal toroid (as if the toroid were made from an ideal solenoid). The direction of the magnetic field within a toroid follows from our curled–straight right-hand rule: Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.