Notes

UNIT 9 ALGEBRA AS GENERALIZED ARITHMETIC

Structure

9.0 INTRODUCTION

You are quite familiar with Arithmetic, a branch of Mathematics. It deals with concrete numbers like 1, 2, 25, 37, 456,…. and various operations on the numbers like addition, subtraction, multiplication, division. But instead of numbers if we use symbols like letters for indicating quantity and perform different arithmetic operations with these letters as was done with the numbers, we are generalizing Arithmetic and name it as Algebra.

Thus, Algebra is a branch of Mathematics where the principles of Arithmetic are generalized by using letter symbols representing numbers. Algebra is very interesting and useful when we try to express a quantity without necessarily attaching any numerical value and solving problems with the operations as we used in Arithmetic.

The Origin of Algebra

The word "Algebra" is derived from an Arabic word 'al-jabar' means reunion, used in a mathematical treatise entitled "Al-Kitab al-mukhta ? ar fi hisab al-gabar wa'l-muqabala" (Arabic for "The compendious Book on calculation by completion and Balancing") written by the Persian Mathematical Muhammed ibn Musa al Khwarizmi of Baghdad in 820 A.D.

The famous Greek mathematician Diophantus living in Alexandria in 3^{rd} century A.D. is regarded as the "Father of Algebra" for his seminal work entitled "Arithmetica".

For completing this unit you will need approximately 7 *(seven) study hours*.

9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to :

- Explain algebraic terms, expressions and categorize algebraic expressions;
- Differentiate between variable and constant, like and unlike terms;
- Perform different operations on algebraic expressions;
- Solve linear algebraic equations in one variable;
- Apply algebraic methods in solving mathematical problems.

9.2 USING SYMBOLS FOR NUMBERS

The main feature of Algebra is to use symbols to represent numbers, quantities or mathematical relations in a general situation rather than only in a particular case as in Arithmetic. Use of letters will allow us to write rules and formulae in a general way. Consider the following examples:

Ex.1. Ayesha has 3 pens and her brother Arvin has 2 pens. So they both have $3 + 2 = 5$ pens.

Notes

In this example, numbers are involved in representing the quantities and in the calculation process. Let us suppose that Ayesha has x number of pens and Arvin has y no. of pens. Can we find the total no. of pens they have? We can say that they have $(x + y)$ no. of pens

Here x and y represent two definite numbers.

Ex. 2. (a) Fig 9.1 is a square whose length of each side is $2\ cm$.

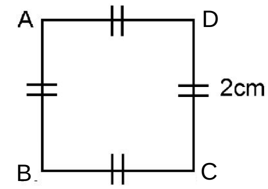


Fig. 9.1

Its perimeter $= AB + BC + CD + DA$

$= (2 + 2 + 2 + 2)\ cm$

$= 2 \times 4$ cm $= 4 \times 2$ cm $= 8$ cm

(b) Find out the perimeter of the square in fig 9.2

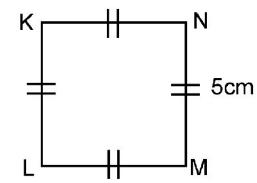


Fig. 9.2

Perimeter of $KLMN$ $= KL + LM + MN + KN$

$= (5 + 5 + 5 + 5)$ cm

$= 5 \times 4$ cm $= 4 \times 5$ cm $= 20$ cm

Thus we can find out the perimeter of any square of given length.

From the above example we can conclude that the perimeter of a square is 4 times the length of each side of it.

Perimeter of square = $4 \times$ length of a side.

If the length of a sides of a square is 'a', then its perimeter 'P' can be expressed as

$$\boxed{P = 4 \times a}$$

Here 'a' represents the number that denotes the length of a side of a particular square and we can find out the perimeter of any square for different values of a.

Thus, with the help of symbols as representation for numbers, a number relation can be generalized. Letters a, b, c ….. x, y, z are used as symbols to represent numbers which are unknown quantities. As a result, various word problems can be expressed as symbolic statements. Since letters represent nos. they follow all the rules and properties of the four arithmetic operations.

9.3 ALGEBRAIC TERMS AND EXPRESSIONS

9.3.1 Algebraic Expressions

In arithmetic we have come across expressions like

$(3 \times 8) + 2$; $(10 \div 5) + (3 \times 20) - 7$ etc.

Notes

In these examples we can observe :

(i) Expressions are formed from nos.

(ii) All the *4* fundamental operations such as addition, subtraction, multiplication and division or some of them are used in an expression

We can form an expression using variables also.

Let us discuss the following examples :

Ex. 3: Babulu is in Class-VI. In his class there are '*m*' girl students. Number of boys is *7* less than the girls. Calculate the total no. of students in his class.

No. of girl students = m

No. of boy students = $m - 7$

Total no. of students = $m + (m - 7) = 2m - 7$

Here $2m - 7$ is an expression which is formed using variable m and constant *2* and *7*. Subtraction and multiplication operations are used here.

$2x + 3$ is an expression, variable x and constants *2* and *3* are used in forming the expression. Addition and multiplication operations are also used.

The expressions, as we got in both the examples above are called algebraic expressions since both variables and constants are used in their formation.

So, a combination of constants and variables connected by some or all of the four fundamental operations +, -, × and ÷ is called an **algebraic expression**.

For example : $7m$, $2py + 1$, $\frac{a}{2} - 5$, $m + n - 2$ are algebraic expressions.

We can write an expression when instruction about how to form it is given. Now observe the example and fill up the table.

Instruction	Expression
16 added to p	P + 16
25 subtracted from r	
P multiplied by (-6)	
X divided by 3	
'm' multiplied by 3 and 8 is added to the product	

Also, when an algebraic expression is given we can tell how it is formed. Now read the example and fill up the blank boxes

Expression	How it is formed
s – 1	1 subtracted from s
t + 25	
11a	
$\frac{2b}{5}$	
2n – 4	

We have seen that the expression have one or more than one term. An algebraic expression is classified into different categories depending upon the no. of terms contained in it.

Monomial : An expression which contains only one term is called a monomial. For example: $7xy, 2x, -4n, -8, 3a^2b$

Binomial : An expression which contains two unlike terms is called a binomial. For example: $x + y$, $2p – 3q$, $z + 1$, $3xy + 2x$

Trinomial : Expression having 3 unlike term is called a trinomial. For example: $2a – 5b + 3c$, $x + y – 3$, $pq + p – 2q$

Polynomial : An expression with one or more terms is known as a polynomial in general. For example: $5x$, $2a + 3b$, $m + n – 3$

Do you know ?

3xy is not a binomial. It is rather a monomial

$m + n – 3$ is not a binomial. It is a trinomial.

$2a + 3a$ is not a binomial. Here the terms are not unlike term.

Monomial, binomial, trinomial are all polynomial.

9.3.2 Variables and Constants

Let us examine mathematical statement $P = 4a$

Here, when $a = 1$, then $p = 4 \times 1 = 4$

when $a = 2$, then $p = 4 \times 2 = 8$

when $a = 3$, then $p = 4 \times 3 = 12$

Thus, we see that for different values taken for *'a'*, the value of *'p'* changes. I.e. *P* varies with the change in the value of a. We say that *'a'* and *'p'* both are changeable or variable. Hence we can say:

> *A symbol which does not have any fixed value for it, but may be assigned any numerical value according to the requirement is known as a **variable**.*

Well, can the number of sides of a triangle be anything other than 3 ? Definitely not. Therefore, the number of sides of a triangle is a fixed number and thus it is a constant. Thus we can say:

> *A symbol having a fixed numerical value is called a **constant**.*

In the statement $P = 4a$, *'a'* and *'p'* are called variables and '4' is a constant.

Do you know ?

A variable has no fixed value.

The letters x, y, z, p, q, r are usually taken to represent variables.

All real nos. are constant.

Algebraic expressions are formed from variables and constants.

Let us discuss the following examples :

Ex. – 5: Papulu purchased 2 similar pens costing 10 rupees each. How much he has to pay to the shopkeeper?

Clearly the cost of *2* pens = *Rs.* 10×2 = *Rs. 20.*, which Papulu has to pay

Here we have, Total cost = cost of each item × no. of items

If we take c for total cost and n for no. of items, the above statement can be written as $C = 10n$

Here n and c both are variables and *10* is a constant.

Ex. – 6: Esma and Reshma are sisters. Esma is older than Reshma by *4* years. Now read the table and fill the blank boxes.

Reshma's Age in Years	Esma's Age in Years
7	7 + 4 = 11
9	
x	

Your answer in the last box will be $(x + 4)$ means Esma's age will be $(x + 4)$ years when Reshma's age is *'x'* years. Here $(x + 4)$ is an algebraic expression where x is a variable and *4* is a constant.

Now write the variables and constant involved in the following expressions :

Expression	Variables	Constant
$y - 7$		
$\frac{s}{2} + 3$		
$2p + 3q$		

9.3.3 Terms of an Algebraic Expression

From earlier discussion we know that an algebraic expression consists of one or more terms. Let us consider the following examples :

Ex. 7 $2p + 3$ is an expression.

In forming the expression we first formed $2p$ separately as a product of 2 and p and then 3 is added to it.

Ex. 8 $xy + 3z - 5$ is an expression.

To form this expression we first formed xy separately as a product of x and y. Then we formed $3z$ separately as a product of 3 and z. We then added them *(xy and 3z)* and then added *(-5)* to it to get the expression.

You will find that the expressions have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are called as terms.

In Example 7, 2p and 3 are terms and in Example 8, xy, 3z and 5 are terms

> *The different parts of the algebraic expression separated from each other by the sign* **'+'** *or* **'-'** *are called the* **terms** *of the expression.*

Do you know?

Terms are added to form expressions.

The sign before a term belongs to the term itself.

Can you find the terms and their number in the following expressions :

Expression	No. of term	Name of the term
2ab – 3	2	2ab, -3
$\frac{k}{3}+1$		
$\frac{-xyz}{2}$		
$16 - x + 3y^2$		

Try this

Write two expressions each having four terms.

9.3.4 Product, Factor and Coefficient

We know from the previous chapters that in the multiplication $2 \times 5 = 10$, 10 is the product and 2 and 5 are the factors of 10.

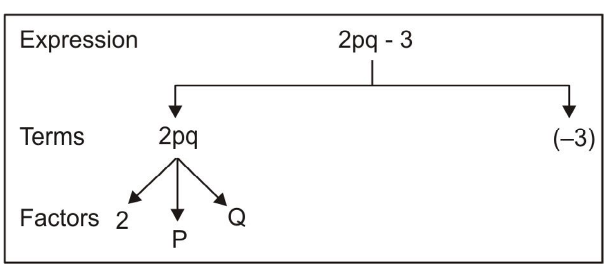
When two variables are multiplied, what is the product ?

We write the product of 3 and $z = 3 \times z = 3z$

And product of y and $z = y \times z = yz$

We saw above that an expression consist of one or more terms. For example expression $2ab - 3$ has 2 terms namely $2ab$ and -3. Here $2ab$ is the product of 2, a and b. We say that 2, a and b are the factors of the term $2ab$.

We can represent any algebraic expression into its constituent terms and terms into the factors by a tree diagram.



Try these : Draw the tree diagram for

(i) 3xy + 5y

(ii) 7ab – 5a + 2

Notes

Do you know ?

A constant factor is called a **numeric** factor.

A variable factor is called is a **literal (algebraic)** factor.

Ex.- 9: The expression 3xy-5y has two terms 3xy and -5y. The numerical factor of a term calls it numerical coefficient or simply coefficient. 3 is the coefficient of 3xy and -5 is the coefficient of y.

Try these : Identify the coefficient of the terms in the following expressions:

(i) -6ab (ii) $-\frac{pq}{3}$

9.3.5 Like and Unlike Terms

Let us examine the factors of the terms $2pq. -pq, 5pq, \left(\frac{1}{2}\right)pq$. These terms have same literal (algebraic) factors pq but different numerical factors. But the terms $2p, 3pq, -5q$ have different literal factors. We say the terms having the same algebraic factors as **similar or like terms**. When the terms do not have the same algebraic factors are called **unlike terms**.

Ex - 1: In the expression $2a + 5ab - 3a - b$, the terms $2a, -3a$ have same algebraic factor a. So they are like terms. But the terms 2a, 5ab have different algebraic factors, they are unlike terms. Similarly the terms $5ab$ and -4 are unlike terms.

Try these :

1. Group the like terms together from. : 7x, 7, -8x, 8y, x, -y, 15y
2. Fill up the table for each group.

Terms	Factors	Algebraic Factor Same or different	Like/Unlike Term
15x 12y	15, x 12, y	Different	unlike
9z –13z			
6xy 2x			
–2ab 5ba			

Notes

Check Your Progress :

Now try to assess your understanding of the concepts covered so far by answering the following questions:

E1. Write any 2 algebraic expressions using the variables p and q.

E2. Write down the expression in the following cases.

(i) 3 added to 5 times the product of x and y

(ii) Sum of a and b subtracted from their product.

E3. Identify the variables and constant in $2y - 3z + 5$

E4. Write down the coefficient of x in each term.

(i) $-x$

(ii) $2xy+y2+z2$

(iii) $\frac{2}{3}x^2y$

E5. Identity the like terms in the following:

$2p$, $-3pq$, $\frac{1}{2}pqr$, -5, $\frac{p}{3}$, $3pqr$, $5pq$

E6. Write the factors in each of the following monomials:

(i) $5xy$

(ii) $-3abc$

E7. Identity the terms and factors in each expression:

(i) $3xy - 5y$

(ii) $ab + 2a - 3y$

9.4 OPERATION ON ALGEBRAIC EXPRESSIONS

We are acquainted with the four fundamental operations – addition, subtraction, multiplication and division on numbers. Here we will move systematically from arithmetic to algebra and learn how these operations are worked on letters representing numbers. Since letters represent nos. they follow all the rules and properties of addition, subtraction, multiplication and division of number.

We will perform different operations in algebra in two phases :

(i) Operations on letters

(ii) Operations on expression

9.4.1 Addition

There are a number of real life problems in which we need to use algebraic expressions and apply arithmetic operations on them. Let us see how expressions are added.

(a) Addition of letters/monomials

We have learnt that $2 + 2 + 2 = 2 \times 3 = 3 \times 2$

similarly we can have $x + x + x = x \times 3 = 3 \times x = 3x$

and $x + x + x + x + x = 5 \times x = 5x$

Now sum of 3x and $5x = 3x + 5x$

$= (x + x + x) + (x + x + x + x + x)$

$= x + x + x + x + x + x + x + x = 8x$

Also $3x + 5x = (3 \times x) + (5 \times x)$

$= (3 + 5) \times x$ (distributive law)

$= 8 \times x = 8x$

Let us see the following example :

Ex-10: Find the sum of $5ab$, $7ab$ and ab

Solution : The sum $= 5ab + 7ab + ab$

$= (5 \times ab) + (7 \times ab) + (1 \times ab)$

$= (5 + 7 + 1) \times ab = 13 \times ab = 13ab$

So, $5ab + 7ab + ab = 13ab$

Try these :

Find the following sum.

(i) $3p$, p and $7p$

(ii) $6xyz$ and $12xyz$

Thus we know how two or more like terms can be added. Now think of the addition of unlike terms.

Let us find the sum of *5* mangoes and *3* oranges.

We can not say that the sum is *8* mangoes or *8* oranges

Similarly, the sum of $5x$ and $3y$ can not be a single term. We write the result as $5x + 3y$ where both the terms are retained.

Notes

(b) Addition of Algebraic Expressions : Let us discuss the following examples :

Ex-11: Find the sum of 5a + 7 and 2a – 5.

The sum $= 5a + 7 + 2a - 5$

$= (5a + 2a) + (7 - 5)$ (putting the like terms together)

$= 7a + 2$

Ex-12: Add $4x + 3y$, $8 + 2x$ and $2y - 5$

The sum $= 4x + 3y + 8 + 2x + 2y - 5$

$= (4x + 2x) + (2y + 3y) + (8 - 5)$ (rearranging terms)

$= 2x + 5y + 3$

Try these :

Add the expressions : (i) $mn + 5$ and $2nm - 7$ (ii) $2a + 3b - 1; 3a + 7$ and $5b - 3$

Note : Expressions obey closure, commutative, associative, properties in addition. It also has additive identify and additive inverse

9.4.2 Subtraction

We know how to subtract integers earlier. The same principle also works with algebraic expressions.

(a) Subtraction of Monomials : Let us subtract $2x$ from $5x$

$5x - 2x = (x + x + x + x + x) - (x + x)$

$= x + x + x + x + x - x - x$

$= x + (-x) + x + (-x) + x + x + x$ [As x and –x are additive

$= 0 + 0 + x + x + x$ inverse of each other]

$= 0 + 3x = 3x$

In brief, we can also do the work as follows:

$5x - 2x = 5 \times x - 2 \times x$

$= (5 - 2) \times x = 3 \times x = 3x$

Let us see another example:

Ex-13 Subtract $7mn$ from $16mn$.

$16mn - 7mn = 16 \times mn - 7 \times mn = (16 - 7)mn = 9 \times mn = 9mn$

But the difference of two unlike terms is not a monomial rather it will be a binomial. For example, difference of $5x$ and $3y = 5x - 3y$

Try these :

Subtract

(i) 5 m from 11 m, (ii) 6ab from 10ab (iii) 5xy from 3xy

(b) Subtraction of Algebraic Expressions: The process of subtraction is similar to that of addition. Let us observe the following examples.

Ex-14 Subtract $3a + 2b$ from $4a + 5b - 2$

Solution: $(4a + 5b - 2) – (3a + 2b)$

$= 4a + 5b - 2 – 3a - 2b$

$= (4a-3a) + (5b-2b) – 2$ (Putting the like terms together)

$= a + 3b – 2$

Alternative method **:** We shell write the expressions one below the other with the like terms remaining in one column and perform subtraction on each of the terms separately as shown below:

$$\begin{array}{rl} & 4a + 5b - 2 \\ - & 3a + 2b \\ \hline & a + 3b - 2 \\ \hline \end{array}$$

Try these :

Subtract (i) $5x – 9$ from $10x + 7b - 3$

(ii) $4pq – 5p + 2$ from $6pq - 1 + 3q$

9.4.3 Multiplication

(a) Multiplication of monomials

$a \times a = a^2$ where 2 is the number that represents the number of a's.

The number 2 in a^2 is known as **index** or **exponent** or **power** of a and 'a' is the **base**.

Try to fill up the table

No. of a's	Product	Base	Index
Product of 3 a' s	*a3*	*a*	*3*
$a \times a \times a \times a$			
$a \times a \times a \times a \times a$			

Notes

Let us learn to multiply two terms.

The product of a and b, that is $a \times b$ can briefly be written as '*ab*'

Similarly, $a \times a \times b = a^2b$

$a \times a \times b \times b = a^2b^2$ and so on.

What is (i) $x \times x \times x \times y \times y =$ ....

(ii) $m \times m \times m \times n \times n \times n =$ ....

Now let us discuss some more examples.

Ex-15 Multiply *2x* by *3y*

$2x \times 3y = 2 \times x \times 3 \times y$

$= 2 \times 3 \times x \times y = 6\,xy$ *(by commutativity of multiplication)*

Ex-16 Multiply *3mn* by *-5mn*

$3mn \times (-5mn) = 3 \times m \times n \times (-5) \times m \times n$

$= 3 \times (-5) \times (m \times m) \times (n \times n)$

$= -15 \times m2 \times n2 = -15\, m^2n^2$

Ex-17 Find the product of *-5pq*, *4pqr* and *2r*

$(-5pq) \times (4pqr) \times 2r = (-5) \times p \times q \times 4 \times p \times q \times r \times 2 \times r$

$= (-5) \times 4 \times 2 \times p \times p \times q \times q \times r \times r$ (rearranging the factors)

$= -40 \times p2 \times q2 \times r2 = -40\, p2\, q2\, r2$

Try these :

Find the product

(i) $4xy \times 2x^2$

(ii) $5m \times 3n \times 7mn$

(b) ***Multiplication of a monomial with a polynomial***

For this multiplication commutative, associative and distributive properties are used as and when required.

Ex-18 Multiply : $(3x - 5)$ with $2x$

Product $= (3x - 5) \times 2x$

$= 3x \times 2x - 5 \times 2x$ (distributive law)

$= 3 \times 2 \times x \times x - 5 \times 2 \times x$

$= 6x^2 - 10x$

Notes

Ex-19 Multiply $3a$ with $(5a - 2b + 4)$

Product $= 3a \times (5a - 2b + 4)$

$= 3a \times 5a + 3a \times (-2b) + 3a \times 4$ (distributive Law)

$= 3 \times a \times 5 \times a + 3 \times a \times (-2) \times b + 3 \times a \times 4$

$= 3 \times 5 \times a \times a + 3 \times (-2) \times a \times b + 3 \times 4 \times a$

$= 15a2 - 6ab + 12a$

Try these :

Find the product of

(i) $2x - 3y$ *and* $5xy$

(ii) $3mn$ *and* $(5m - 7mn + 3n)$

(c) ***Multiplication of a polynomial by a polynomial*:**

Here also we use distributive property.

Ex-20 Multiply $(a + b)$ by $(3a - 5b)$

Solution : $(a + b)(3a - 5b)$

$= a(3a - 5b) + b(3a - 5b)$ (distributive Law)

$= a \times 3a - a \times 5b + b \times 3a + b \times (-5b)$

$= 3 \times a \times a - 5 \times a \times b + 3 \times a \times b - 5 \times b \times b$

(associative and commutative property)

$= 3a^2 - 5ab + 3ab - 5b^2$

$= 3a^2 - 2ab - 5b^2$

Ex-21 Multiply $(2x + 5)$ by $(x^2 - 3x + 2)$

Solution **:** $(2x + 5) \times (x2 - 3x + 2)$

$= 2x \times (x2 - 3x + 2) + 5(x2 - 3x + 2)$ (distributive Law)

$= 2x \times x2 - 2x \times 3x + 2x \times 2 + 5 \times x2 - 5 \times 3x + 5 \times 2$

$= 2x3 - 2 \times 3 \times x \times x + 2 \times 2x + 5x2 - 15x + 10$

$= 2x3 - 6x2 + 5x2 + 4x - 15x + 10$

$= 2x3 + (-6 + 5)x2 + (4 - 15)x + 10$

$= 2x3 - x2 - 11x + 10$

Notes

Try these :

Find the poduct of

(i) $(2m + 3n)$ *and* $(m-2n)$

(ii) $(pq-1)$ *and* $(2p + 3q - 5)$

9.4.4 Division

You know the procedure for division of numbers. Similar procedure is also followed while dividing an algebraic expression by another

(a) ***Division of monomial by a monomial:***

Algorithm :

(i) Write the dividend as numerator and divisor as denominator.

(ii) Express the numerator and denominator both as the product of factors.

(iii) Simplify the fraction by cancelling the common factors from the numerator and denominator.

Now observe the following examples:

Ex-22

(i) Division of 15mn by 5m $= \dfrac{15mn}{5m} = \dfrac{3\times5\times m\times n}{5\times m} = 3n$

(ii) $18x^2y^2 \div (-6xy) = 18\dfrac{x^2y^2}{-6xy} = \dfrac{3\times6\times x\times x\times y\times y}{-6\times x\times y} = -3xy$

Try these :

Divide (i) $25xy$ by $-5y$ (ii) $30a^2b^2c$ by $6ab$

(b) ***Division of polynomial by a monomial:***

Here the dividend is a polynomial and the divisor is a monomial. The working rule for division is :

Divide each term of the dividend by the divisor.

Simplify each fraction as earlier.

Let us workout the following examples:

Ex-23

(i) Divide $9x^2 - 15xy$ by $3x$

$$Now,\ (9x^2 - 15xy) \div (3x) = \frac{9x^2 - 15xy}{3x} = \frac{9x^2}{3x} - \frac{15xy}{3x} = 3x - 5y$$

(ii) Divide $8a^2b - 12ab^2 + 20ab$ *by* $(-4ab)$

Now, $(8a^2b - 12ab^2 + 20ab) \div (-4ab) = -2a + 3b - 5$

Try these :

Divide (i) $6m^2n - 9mn^2$ *by* $3mn$

(ii) $10x^3y - 15x^2y^2 - 5x^2y^3$ *by* $5xy$

(c) *Division of Polynomial by a Polynomial:*

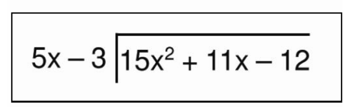
In case of numbers, we know how to divide the dividend by a divisor by the long division process. we have to follow the similar procedure here.

Consider the following examples.

Ex-24: Divide $11x + 15x^2 - 12$ by $5x - 3$

Step-1

Arrange the terms of the dividend and dvisor in descending order of powers of a certain variable contained in the polynomials.



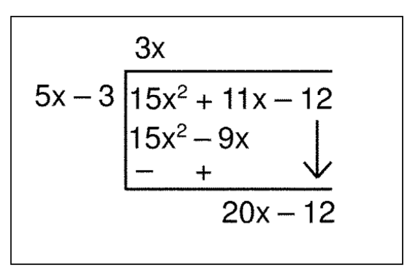
Step-2

Divide the 1st term of the dividend by the 1st term of divisor and get the 1st term of the quotient.

Here $\frac{15x^2}{5x} = 3x$

Step-3

Multiply $3x$ with each term of the divisor and write the result below the dividend. Then subtract it from the dividend.



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Step-5

Repeat the process from step 2 to 3 considering $20x - 12$ as the new dividend.

$$\begin{array}{r|l}
 & 3x + 4 \\
\hline
5x - 3 & 15x^2 + 11x - 12 \\
 & 15x^2 - 9x \downarrow \\
 & - \quad + \\
\hline
 & \qquad 20x - 12 \\
 & \qquad 20x - 12 \\
 & \qquad - \quad + \\
\hline
 & \qquad\qquad 0
\end{array}$$

Here the 2nd term of the quotient $= \dfrac{20x}{5x} = 4$

Product of 2nd term of the quotient and the divisor is now to be subtracted from the new dividend. Result is '0'.

Hence the quotient = 3x + 4 and the remainder = 0

$$\therefore \quad \left(15x^2 + 11x - 12\right) \div (5x - 3) = 3x + 4$$

Thus we can divide a polynomial by another polynomial in the above mentioned long division process.

Note : The 1st term of the dividend should not be of a lower power than the 1st term of the divisor.

Now try to Check Your Progress:

E8. Simplify: $5x^2 - 6xy - y^2 - 2x^2 - 3y^2 + 2xy - 2y^2 + x^2$

E9. Subtract $3m - 5n + 7$ from the sum of $2m - 3n + 5$ and $8 + 4n$.

E10. Fill in the blank space:

$$\left(a^2 - 5ab - 3a + 7\right) + (................) = 3a^2 + 2ab - 5b + 2$$

E11. Multiply

(a) $(3x - 2)$ by $(2x + 3)$

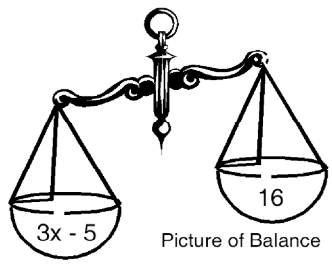
(b) $\left(p^2 + pq + q^2\right)$ by $(p - q)$

E12. Divide $3a^3 + 16a^2 + 20 + 21a$ by $(a + 4)$.

9.5 LINEAR ALGEBRAIC EQUATION AND ITS SOLUTION

An equation can be compared with a balance in equilibrium. The two sides of the balance being compared with the two sides i.e. LHS and RHS of an equation. The equality sign indicates that the scale pans are balanced. For example in the equation

$3x - 5 = 16$, the LHS $3x - 5$ is in the left pan and RHS 16 is in the right pan of the balance. Also the balance is in equilibrium as the values of the two sides of the equation are equal. x in the equation is known as the unknown.



Picture of Balance

In the sub-unit the most basic form of algebraic equations i.e. the linear equation will be extensively discussed. The process of solving these equations will also be discussed.

9.5.1 Linear Algebraic Equation

Let us discuss a simple problem, 'to find a number which, when doubled and then added to *5* gives *15*.'

To write the mathematical statement, in this case, we take the unknown as x, then double it and obtain $2x$. When *5* is added to this, the result is $2x + 5$. We know that this equals to *15*. Hence the mathematical statement is $2x + 5 = 15$, which is the equation in the above case.

When a polynomial is formed using a certain symbol (taken for an unknown) and is equated to a certain number, the statement formed is an **equation**.

So, $2x + 5 = 15$ is one of the simple forms of algebraic equations. Let us observe the example and write the equation in the table.

Statement	Equation
Sum of a number **x** and 7 is 16	$x + 7 = 16$
3 subtracted from **y** is 10	
9 times of **n** is 36	
Three – fourth of **m** is 12	

The equations you write in the above table are known as a linear equation with one unknown and the power of unknown is 1. Hence we say :

An equation involving only a linear polynomial is called a **linear equation**. A linear equation in one variable generally is of the form $ax + b = 0$ where a, b are numbers and x is unknown

Linear equations are also known as **first degree** equations. We shall restrict our present discussion to such equations only.

Notes

Note :

The equation $x^2 + 7x + 6 = 0$ is not linear because the exponent of the unknown is *2*.

The equation $x + 2y = 5$ is linear but it contains 2 unknowns x & y.

9.5.2 Solution of Linear Equations

Consider the linear equation $x + 7 = 16$. Now what is the value of x ? We will go on trying the integer *1, 2, 3………* for 'x' till we get both sides equal. It can be seen by trial that only when x = *9*, at that point, equality of both sides of the equation occur. For no other value of x the equation is satisfied.

Thus we know that the equation is satisfied i.e. both the sides have equal value only for one value of the unknown.

For example, the equation $2y = 6$ is satisfied only for y = 3. Similarly the equation $m – 3 = 4$ is satisfied only for $m = 7$.

The value of the unknown which makes the equation a true statement is called the **solution** or **root** of the equation.

To solve an equation is to find the value of the unknown which satisfies the equation.

Process of Solving a Linear Equation :

(a) Process of Trial :

Let us complete the table and by inspection of the table find the solution to the equation x + 7 = 16.

x	1	2	3	4	5	6	7	8	9	10
x + 7										

Here we go on trying with the integers for 'x' till the equation holds good. In this way we can obtain the value of the unknown. We say this process as the **process of trial**. You have seen that after so many unsuccessful trials we got the value of the unknown which satisfies the equation. Hence, this process is tedious, time consuming and often we fail to find the solution if it is a large number.

Try these :

Solve the equation by process of trial.

(i) m – 5 = 16

(ii) 2y – 1 = 17

(b) Process of Adding or Subtracting :

An equation, as already said can be compared to a balance in equilibrium. The two sides of the equation are like the two pans and the equality sign indicates that the pans are balanced. Doing an arithmetic operation on an equation is like adding weights or removing weights from the pans of the balance. If we add the same weights to both the pans the beam of the balance remains horizontal. Similarly, if we remove equal weights from the pans, the beam also remains horizontal. On the other hand, if we add or remove different weights, the balance is tilted, that is the beam of the balance does not remain horizontal. We use this principle for solving an equation.

Supposing 'x' represents the weight of packet of rice kept on the left pan and a weight 'w' kept on the right pan and the balance remains in equilibrium, then we say : x = w.

Case – I

If we add a weight 'c' on both the pans, whether the beam remains horizontal or tilted ?

Surely it will remain horizontal.

Thus we get, $x + c = w + c$

Case – II

Had we removed weight 'c' from both of the pans, in what state would the beam remain ?

Definitely horizontal.

Thus we get $x - c = w - c$

Case – III

Similarly, if we make the weights on both the pans 'c' times, the beam would still remain horizontal.

The mathematical representation of this situation is

$xc = wc$

Case – IV

Also if we make the weights on both the pans '$\frac{1}{c}$', (where $c \neq o$) times then the beam would remain balanced.

Thus we get, $\frac{x}{c} = \frac{w}{c}$ (when $c \neq 0$)

From the above properties of weighing with a balance, we got four rules of equality for solving an equation. Those rules help in solving linear equations in precise manner. The rules state as follows :

The same quantity can be added to both sides of an equation and it does not change the equality.

The same quantity can be subtracted from both sides of an equation and it does not change the equality.

Both sides of an equation may be multiplied by the same number and it does not change equality.

Both sides of an equation may be divided by a non-zero number and it does not change the quality.

Let us apply the above rules in solving the linear equations :

Ex-25 Solve: $y - 5 = 11$

Solution : Solving an equation means finding the value of the unknown.

So, $y - 5 = 11$

$\Rightarrow (y - 5) + 5 = 11 + 5$ (Adding 5 on both the sides)

$\Rightarrow y + (-5 + 5) = 16$ (Associativity)

$\Rightarrow y + 0 = 16$ (Additive inverse)

$\Rightarrow y = 16$ (Additive identity)

Ex-26 Solve $z + 4 = 8$

Solution : $z + 4 = 8$

$\Rightarrow (z + 4) - 4 = 8 - 4$ (Subtracting 4 from both the sides)

$\Rightarrow z + (4 - 4) = 4$ (Associativity)

$\Rightarrow z + 0 = 4$ (Additive inverse)

$\Rightarrow z = 4$ (Additive Identity)

Ex-27 Solve : $\frac{x}{3} = 12$

Solution : $\frac{x}{3} = 12$

$\Rightarrow \frac{x}{3} \times 3 = 12 \times 3$ (Multiplying 3 to both sides)

$\Rightarrow x \times \left(\frac{1}{3} \times 3\right) = 36$ (Associativity)

$\Rightarrow x \times 1 = 36$ (Multiplicative inverse)

$\Rightarrow x = 36$ (Multiplicative identity)

Ex-28 Solve : $5x - 2 = 28$

Solution : Here the numerical term (constant) is to be eliminated first.

So, $5x - 2 = 28$

$\Rightarrow (5x - 2) + 2 = 28 + 2$ (Adding 2 to both the sides)

$\Rightarrow 5x + (-2 + 2) = 30$ (Associativity)

$\Rightarrow 5x + 0 = 30$ (Additive inverse)

$\Rightarrow 5x = 30$ (Additive identity)

$\Rightarrow \frac{5x}{5} = \frac{30}{5}$ (Dividing both sides by 5)

$\Rightarrow x = 6$ (Answer)

Note :

In practice we shall not show the steps where the rule/properties are applied.

Try these :

Solve the following equation :

(i) $3x + 2 = 14$ *(ii)* $\frac{x}{4} - 1 = 5$

(c) Process of Transposition

While solving a linear equation, we can transpose a number from LHS to RHS or vice versa instead of adding or subtracting it from both sides of the equation. In doing so, the operation that connects the number on one side of the equation changes as it is removed to the other side. That is on transposing a term from one side to the other the operation of :

(i) Addition changes to subtraction

(ii) Subtraction changes to addition

(iii) Multiplication changes to division

(iv) Division changes to multiplication

These are called the **Rules of Transposition**.

Let us apply these rules in solving the following equations.

Ex-29 Solve: $2x - 7 = 5$

Solution: $2x - 7 = 5$

$\Rightarrow 2x = 5 + 7$ (Transposing 7 to RHS)

$\Rightarrow 2x = 12$

$\Rightarrow x = \frac{12}{2}$ (Transposing 2 to RHS)

$\Rightarrow x = 6$

Ex-30 Solve : $\frac{5y - 2}{3} = 6$

Solution: $\frac{5y - 2}{3} = 6$

$\Rightarrow 5y - 2 = 6 \times 3$

$\Rightarrow 5y - 2 = 18$

$\Rightarrow 5y = 18 + 2$

$\Rightarrow 5y = 20$

$\Rightarrow y = \frac{20}{5} = 4$

$\therefore y = 4$

Try these :

Solve by transposition

(i) $3p + 2 = 17$ (ii) $2(x + 4) = 12$

(d) Rule of Cross Multiplication

If the equation involves a fraction, let us learn an easier method to remove the fraction without disturbing the equality.

Suppose the equation is of the form $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow a = \frac{c}{d} \times b$ (Transposing 'b' to RHS)

Notes

$\Rightarrow a = \dfrac{c \times b}{d}$

$\Rightarrow a \times d = c \times b$ (Transposing 'c' to LHS)

Thus we find: $\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow a \times d = c \times b$

Otherwise – $\dfrac{a}{b} = \dfrac{c}{d}$

$\Rightarrow \dfrac{a}{b} \times bd = \dfrac{c}{d} \times bd$ (Multiplying 'bd' on both sides)

$\Rightarrow ad = cb$

This is known as Rule of Cross Multiplication.

Let us observe some examples for a better understanding.

Ex-31 Solve : $\dfrac{3x+1}{2} = \dfrac{x+7}{4}$

Solution: $\dfrac{3x+1}{2} = \dfrac{x+7}{4}$

$\Rightarrow (3x+1) \times 4 = (x+7) \times 2$ (By cross multiplying)

$\Rightarrow 12x + 4 = 2x + 14$ (Distribute law)

$\Rightarrow 12x - 2x = 14 - 4$ (Bringing unknown terms to LHS and Constants to RHS)

$\Rightarrow 10x = 10$

$\Rightarrow x = \dfrac{10}{10}$ (Transposing 10 to RHS)

$\Rightarrow x = 1$

Ex-32 Solve : $\dfrac{3y-1}{2y+3} = \dfrac{5}{7}$

Solution: $\dfrac{3y-1}{2y+3} = \dfrac{5}{7}$

$\Rightarrow (3y - 1) \times 7 = (2y + 3) \times 5$ (cross multiplication)

$$\Rightarrow 21y - 7 = 10y + 15$$

$$\Rightarrow 21y - 10y = 15 + 7$$

$$\Rightarrow 11y = 22$$

$$\Rightarrow y = \frac{22}{11} = 2$$

Thus we have discussed four processes of solving a linear equation with one unknown. Among those, processes of transposition and cross multiplication are widely used.

Now try to answer the following questions to Check Your Progress:

E13 Solve the following equations.

(i) $7x = 28$ (ii) $3y - 2 = 19$ (iii) $\frac{x}{2} - 3 = 6$

E14 Solve the following equations.

(i) $28 = 4 + 3(t + 5)$ (ii) $2(2p - 3) = 6$

E15 Solve the following equations.

(i) $\frac{8x}{3x+6} = \frac{4}{3}$ (ii) $\frac{2x+3}{3x+7} = \frac{5}{8}$

9.6 SOLUTION OF WORD PROBLEMS/ APPLICATION OF ALGEBRAIC METHODS

We have learnt how to convert a mathematical statement into a simple equation. Also we have learnt how to solve simple equations. The algorithm (method) involved in solving real life problems is to :

(i) Understand the situation expressed in the word problem.

(ii) Choose a symbol and substitute it for the unknown to be determined.

(iii) Write an equation from the given relation in the problem.

(iv) Solve the equation and find the value of the unknown.

(v) Verify the correctness of the solution.

Let us discuss the following examples:

Ex- 33 Mita thinks of a number. If she takes away 7 from 4 times of the number, the result is 17. What is the number ?

Solution :

Suppose Mita thinks of the number x.

4 times of the number = 4x

After taking away 7 from 4x, Mita has 4x - 7.

According to the problem, 4x - 7 = 17

Hence we got the equation for x.

Let us now solve the equation 4x - 7 = 17

$\Rightarrow \; 4x = 17 + 7 \Rightarrow \; 4x = 24$

$\Rightarrow \; x = \dfrac{24}{4}$

$\Rightarrow \; x = 6$

So, the required number is 6

Checking the answer :

LHS = 4x – 7 = 4 × 6 – 7 = 24 – 7 = 17 = RHS as required

Ex-34 Vicky is 5 years older than Ricky. 15 years back Vicky's age was 2 times that of Ricky. What are their present ages ?

Solution :

Among them, Ricky is younger.

So let us take the present age of Ricky to be z

The present age of Vicky = (z + 5) years

15 years back, the age of Vicky was z + 5 – 15 = z – 10 years.

And the age of Ricky then was (z – 15) years.

According to the given relation.

Vicky's age is 2 times of Ricky's age

$\Rightarrow \; z - 10 = 2\,(z - 15)$

$\Rightarrow \; z - 10 = 2z - 30$

$\Rightarrow \; z - 2z = -30 + 10 \qquad \Rightarrow \; z = 20$

So, the present age of Ricky = 20 years and that of Vicky = 20 + 5 = 25 years.

Solve the following to Check Your Progress:

E16. The sum of two numbers is 64. One number is 14 more than the other. What are the numbers ?

E17. Solve the following problems.

(a) Sachin scores twice as many runs as Dhoni. Together their runs fell one short of a century. How many runs did each one score?

(b) People of Naraharipur planted 102 trees in the village garden. The number of non-fruit trees ware 2 more than three times the number of fruit trees. What is the number of fruit trees planted?

(c) Sania's age is half the age of her father and her father's age is half the age of her grand father. After twenty years her age will be equal to the present age of her father. What are the present ages of Sania, her father and her grand father?

E18. Solve the following equation:

$$\frac{2x+3}{3x-7} = \frac{5}{8}$$

9.7 LET US SUM UP

Algebra is generalized arithmetic where letters are used as symbols to represent numbers. Every number is a constant and every symbol can be assigned different values in different situations.

Algebraic expressions are formed using symbols and constants. We also use four fundamental operations on the symbols and constants to from expressions.

Terms are parts of an expression which are separated by '+' or '_' sign. It may be a constant, a variable or combination of both.

Any expression having one or more terms is called a Polynomial. Specifically expression having one term is called monomial, having two terms is a binomial and degree of the polynomial is the highest degree of the term among all the terms

The two sides of an equation are like the two pans of a balance

Linear equations can be solved by using any of the four methods : by trial, addition and subtraction of equal quantity to both the sides, transposition and cross multiplication.

9.8 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1. p + q, 2p – q, 3p + 2q – 1 or any three similar expressions.

E2. (i) 5xy + 3 (ii) ab - (a + b)

E3. Variables = y & z, Constant = 2, 3, 5

E4. (i) -1, (ii) 2y (iii) $\frac{2}{3}$ xy

E5. $\left(2p, \frac{p}{3}\right)$, (-3pq, 5pq), $\left(\frac{1}{2}pqr,\ 3pqr\right)$

E6. (i) coefficient of xy = 5 , that of 5 = xy etc.

(ii) Coefficient of 3 = -abc, that of abc = -3 etc.

E7. (i) Terms = 3xy, 5y ; Factors of 3xy = 3, x, y ; Factors of 5y = 5, y

(ii) Terms = ab, 2a and 3y

E8. 4x2 – 4xy – 6y2

E9. - m+ 6n + 6

E10. 2a2 + 7ab + 3a - 5b – 5

E11. (a) 6x2 + 5x – 6 (b) p3 – q3

E12. (i) 3a2 + 4a + 5

E13. (i) x = 4 (ii) y = 7 (iii) x = 18

E14. (i) t = 3 (ii) p = 3

E15. (i) x = 2 (ii) x = 11

E16. 25 and 39

E17. (a) Dhoni = 33, Sachin = 66 (b) 25 (c) 20 ,40, 80

E18. x = 11

9.9 SUGGESTED READINGS AND REFERENCES

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NCTM (1999). *Activities for Junior High School and Middle School Mathematics, Vol.2*, The National Council of Teachers of Mathematics(NCTM), INC, USA.

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9.10 UNIT-END EXERCISE

Notes

1. Write the expression for the followings.

   (i) One third of the sum of numbers p and q.

   (ii) Sum of the nos. a and b subtracted from their product.

   (iii) 8 added to 2 times the product of x & y.

2. Find the coefficient (i) of xy in – 5xyz, (ii) of m^2 in 2 m^2n^2 (iii) 9 in – 9 pqr.

3. Identify the numerical coefficient of (i) –t (ii) $\frac{2}{3}pq$ (iii) – 8 x2y2

4. Give one example of (i) Monomial (ii) Binomial (iii) Trinomial.

5. Write the like terms together in separate groups

   (i) ab^2, - 4ab, $2a^2b$, ab, - $3ab^2$, $\frac{2}{3}$ab, -$5a^2b$, a^2b^2

   (ii) 2x, - 5xy, - x, $\frac{xy}{2}$, 3y

6. Add the following expressions :

   (i) a + b – 5, b – a + 3 and a – b + 6

   (ii) 4x + 3y – 7xy, 3xy – 2x and 2xy – y

   (iii) $m^2 – n^2 – 1$, $n^2 – 1 – m^2$ and $1 – m^2 – n^2$

7. Subtract :

   (i) x (y – 3) from y (3 – x)

   (ii) m^2 + 10m – 5 from 5m – 10.

   (iii) $5a^2 – 7ab + 5b^2$ from $2ab – 3a^2 – 3b^2$

8. Simplify the expression :

   (i) $10x^2 – 8x + 5 + -5x – 4x^2 – 6m – 10$

   (ii) 20mn – 10n – 17m – 12n + 14m + 2

9 Multiply :

   (i) (a – b) by (a2 + ab + b2)

   (ii) (p + q – 5) by (p - q)

10. Divide:

    (i) $(8m^2 + 4m - 60)$ by $(2m - 5)$

    (ii) $(6\,a^2b^2 - 7abc - 3b^2)$ by $(3ab + c)$

11. Solve the following equations:

    (i) $2y - 5 = 9$

    (ii) $\frac{3}{5} + x = \frac{13}{5}$

    (iii) $\frac{z}{3} + \frac{z}{5} = 40$

    (iv) $\frac{8x}{6 + 3x} = \frac{-4}{3}$

12. Solve the following problems :

    (i) The length of a rectangular plot exceeds its breadth by 5m. The perimeter of the plot is 70m. Find the length of the plot.

    (ii) In an isosceles triangle the base angles are equal. The vertex angle is 15° more than each of the base angles. Find the measure of the angles of the triangle.