1.1. Quadratic Equation

An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a **quadratic equation** or an equation of the **second degree**.

A second degree equation in one variable x of the form

$$ax^2 + bx + c = 0 \quad , \quad \text{where } a \neq 0 \text{ and } a, b, c \text{ are real numbers,}$$

is called the **general or standard form** of a quadratic equation.

Here a is the co-efficient of x^2, b is the co-efficient of x and constant term is c.

The equations $x^2 - 7x + 6 = 0$ and $3x^2 + 4x = 5$ are the examples of quadratic equations.

Remember that: If $a = 0$ in $ax^2 + bx + c = 0$, then it reduces to a linear equation $bx + c = 0$.

$x^2 - 7x + 6 = 0$ is in standard form but

$3x^2 + 4x = 5$ is not in standard form.

If $b = 0$ in a quadratic equation $ax^2 + bx + c = 0$,

then it is called a **pure quadratic** equation. For example $x^2 - 16 = 0$ and $4x^2 = 7$ are the pure quadratic equations.

Activity: Write two pure quadratic equations.

1.2 Solution of quadratic equations

To find solution set of a quadratic equation, following methods are used:

(i) factorization (ii) completing square

1.2(i) Solution by factorization:

In this method, write the quadratic equation in the standard form as

$$ax^2 + bx + c = 0 \qquad \text{(i)}$$

If two numbers r and s can be found for equation (i) such that $r + s = b$ and $rs = a$

then $ax^2 + bx + c$ can be factorized into two linear factors.

The procedure is explained in the following examples.

Example 1: Solve the quadratic equation $3x^2 - 6x = x + 20$ by factorization.

Solution: $3x^2 - 6x = x + 20 \qquad \text{(i)}$

The standard form of (i) is $3x^2 - 7x - 20 = 0 \qquad \text{(ii)}$

Here $a = 3, b = -7, c = -20$ and $ac = 3 \times -20 = -60$

As $-12 + 5 = -7$ and $-12 \times 5 = -60$, so

the equation (ii) can be written as

$$3x^2 - 12x + 5x - 20 = 0$$

or $3x(x - 4) + 5(x - 4) = 0$

$\Rightarrow (x - 4)(3x + 5) = 0$

Activity: Factorize $x^2 - x - 2 = 0$.

Either $x - 4 = 0$ or $3x + 5 = 0$, that is,

$x = 4$ or $3x = -5 \Rightarrow x = -\frac{5}{3}$

$\therefore$ $x = -\frac{5}{3}, 4$ are the solutions of the given equation.

Thus, the solution set is $\left\{-\frac{5}{3}, 4\right\}$.

Example 2: Solve $5x^2 = 30x$ by factorization.

Solution: $5x^2 = 30x$

$5x^2 - 30x = 0$ which is factorized as

$5x(x-6) = 0$

Either $5x = 0$ or $x - 6 = 0 \Rightarrow x = 0$ or $x = 6$

$\therefore$ $x = 0, 6$ are the roots of the given equation.

Thus, the solution set is $\{0, 6\}$.

Remember that: Cancelling of x both sides of $5x^2 = 30x$ means the lo of one root *i.e.*, $x = 0$

1.2(ii) Solution by completing square:

To solve a quadratic equation by the method of completing square is illu through the following examples.

Example 1: Solve the equation $x^2 - 3x - 4 = 0$ by completing square.

Solution: $x^2 - 3x - 4 = 0$ (i)

Shifting constant term -4 to the right, we have

$x^2 - 3x = 4$ (ii)

Adding the square of $\frac{1}{2} \times$ coefficient of x, that is,

$\left(-\frac{3}{2}\right)^2$ on both sides of equation (ii), we get

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 4 + \left(-\frac{3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = 4 + \frac{9}{4} = \frac{16+9}{4}$$

or $$\left(x - \frac{3}{2}\right)^2 = \frac{25}{4}$$

Taking square root of both sides of the above equation, we have

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \pm\sqrt{\frac{25}{4}}$$

$$\Rightarrow \quad x - \frac{3}{2} = \pm\frac{5}{2} \text{ or } x = \frac{3}{2} \pm \frac{5}{2}$$

Either $x = \frac{3}{2} + \frac{5}{2} = \frac{3+5}{2} = \frac{8}{2} = 4$ or $x = \frac{3}{2} - \frac{5}{2} = \frac{3-5}{2} = \frac{-2}{2} = -1$

1. **Write the following quadratic equations in the standard form and point out pure quadratic equations.**

(i). $(x+7)(x-3)=-7$ (ii) $\frac{x^2+4}{3}-\frac{x}{7}=1$

(iii) $\frac{x}{x+1}+\frac{x+1}{x}=6$ (iv) $\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$

(v) $\frac{x+3}{x+4}-\frac{x-5}{x}=1$ (vi) $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

2. **Solve by factorization:**

(i) $x^2-x-20=0$ (ii) $3y^2=y(y-5)$

(iii) $4-32x=17x^2$ (iv) $x^2-11x=152$

(v) $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$ (vi) $\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$

3. **Solve the following equations by completing square:**

(i) $7x^2+2x-1=0$ (ii) $ax^2+4x-a=0\ ,\ a\neq 0$

(iii) $11x^2-34x+3=0$ (iv) $lx^2+mx+n=0,\ l\neq 0$

(v) $3x^2+7x=0$ (vi) $x^2-2x-195=0$

(vii) $-x^2+\frac{15}{2}=\frac{7}{2}x$ (viii) $x^2+17x+\frac{33}{4}=0$

(ix) $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$ (x) $7(x+2a)^2+3a^2=5a\,(7x+23a)$

1.3 Quadratic Formula:

1.3. (i) Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$$ax^2+bx+c=0\quad ,\quad a\neq 0$$

Dividing each term of the equation by a, we get

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

Shifting constant term $\frac{c}{a}$ to the right, we have

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides, we obtain

.3 (ii) Use of quadratic formula:

The quadratic formula is a useful tool for solving all those equations which can or can not be factorized. The method to solve the quadratic equation by using quadratic formula is illustrated through the following examples.

Example 1: Solve the quadratic equation $2 + 9x = 5x^2$ by using quadratic formula.

Solution: $2 + 9x = 5x^2$

The given equation in standard form can be written as

$$5x^2 - 9x - 2 = 0$$

Comparing with the standard quadratic equation $ax^2 + bx + c = 0$, we observe that

$$a = 5\ ,\ b = -9\ ,\ c = -2$$

Putting the values of a, b and c in quadratic formula

Activity: Using quadratic formula, write the solution set of $x^2 + x - 2 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\ , \text{ we have}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(-2)}}{2(5)}$$

or $$x = \frac{9 \pm \sqrt{81 + 40}}{10} = \frac{9 \pm \sqrt{121}}{10} = \frac{9 \pm 11}{10}$$

Either $x = \dfrac{9 + 11}{10}$ or $x = \dfrac{9 - 11}{10}$, that is,

$$x = \frac{20}{10} = 2 \quad \text{or} \quad x = \frac{-2}{10} = -\frac{1}{5}$$

$\therefore\ 2, -\frac{1}{5}$ are the roots of the given equation.

Thus, the solution set is $\left\{-\frac{1}{5}, 2\right\}$.

Example 2: Using quadratic formula, solve the equation $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$.

Solution: $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$

Simplifying and writing in the standard form

$(2x+1)(x+4) - (x-2)(x+2) = 0$

$2x^2 + 8x + x + 4 - (x^2 - 4) = 0$

$2x^2 + 9x + 4 - x^2 + 4 = 0$

or $\quad x^2 + 9x + 8 = 0$

Here $\quad a = 1, b = 9, c = 8$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we have

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{-9 \pm \sqrt{81 - 32}}{2} = \frac{-9 \pm \sqrt{49}}{2} = \frac{-9 \pm 7}{2}$$

$\Rightarrow \quad x = \frac{-9+7}{2} = \frac{-2}{2} = -1$

or $\quad x = \frac{-9-7}{2} = \frac{-16}{2} = -8$

$\therefore\ -1, -8$ are the roots of the given equation. Thus, the solution set is $\{-8, -1\}$.

EXERCISE 1.2

1. **Solve the following equations using quadratic formula:**

(i) $2 - x^2 = 7x$ (ii) $5x^2 + 8x + 1 = 0$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$ (iv) $4x^2 - 14 = 3x$

(v) $6x^2 - 3 - 7x = 0$ (vi) $3x^2 + 8x + 2 = 0$

(vii) $\frac{3}{x-6} - \frac{4}{x-5} = 1$ (viii) $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

(ix) $\frac{a}{x-b} + \frac{b}{x-a} = 2$ (x) $-(l+m) - lx^2 + (2l+m)x = 0, l \neq 0$