

TOPIC: CONCEPT OF GCF AND LCM

GCF:

The greatest common factor (GCF) is the greatest factor that is common to two or more numbers

GCF by Listing out the Factors

Find the GCF of 24 and 36

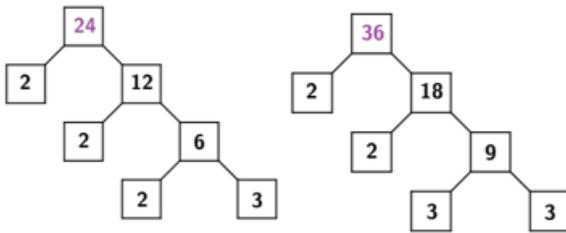
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Greatest Factor common in both numbers is 12

GCF using Prime Factorization

Find the GCF of 24 and 36



Find the prime factors that are common in both numbers

$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$

$$\text{GCF: } 2 \times 2 \times 3 = 12$$

GCF using Repeated Division

Find the GCF of 24 and 36

$$\text{GCF: } 2 \times 2 \times 3 = 12$$

$$\begin{array}{r|rr} 2 & 24 & 36 \\ 2 & 12 & 18 \\ 3 & 6 & 9 \\ & 2 & 3 \end{array}$$

TWO METHOD OF FINDING GCF:

1 - Method that may be used for small numbers:

Examples: Find greatest common factor of the numbers 12 and 18.

- 1) List all the factors of 12 which are: 1, 2, 3, 4, 6, 12
- 2) List all the factors of 18 which are: 1, 2, 3, 6, 9, 18

Look for the common factors of 12 and 18: 1, 2, 3, and 6.

The greatest of the common factor (GCF) of 12 and 18 is 6.

2 - Method that may be used for all numbers including large numbers:

Examples: Find greatest common factor of the numbers 60 and 90.

1) The prime factorization of 60 is: $60 = 2 \times 2 \times 3 \times 5 = 2 \times 2 \times 3 \times 5$

2) The prime factorization of 90 is: $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3 \times 3 \times 5$

Look for the common factors of 60 and 90 in the prime factorization: 2, 3, and 5.

The greatest common factor (GCF) of 60 and 90 is $2 \times 3 \times 5 = 30$.

Sol questions:

Q1: Find the greatest common factor of 36 and 42.

Solution

The prime factorization of 36 and 42 are:

$$36 = 2 \times 2 \times 3 \times 3$$

$$42 = 2 \times 3 \times 7$$

$$\text{GCF of } 36 \text{ and } 42 = 2 \times 3 = 6$$

Q2: Find the greatest common factor of 45, 60 and 75.

Solution

The prime factorization of 45, 60 and 75 are:

$$45 = 3 \times 3 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$75 = 3 \times 5 \times 5$$

$$\text{GCF of } 45, 60 \text{ and } 75 = 3 \times 5 = 15$$

Q3: What is the greatest common factor of 360 and 252?

Solution

The prime factorization of 360 and 252 are:

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

$$252 = 2 \times 2 \times 3 \times 3 \times 7$$

$$\text{GCF of } 360 \text{ and } 252 = 2 \times 2 \times 3 \times 3 = 36$$

Q4: What is the greatest common factor of 324, 666 and 756?

Solution

The prime factorization of 324, 666 and 756 are:

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$$666 = 2 \times 3 \times 3 \times 37$$

$$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$$

$$\text{GCF of } 324, 666 \text{ and } 756 = 2 \times 3 \times 3 = 18$$

Q5: a) Find the GCF of 12 and 16.

b) Use the result in part a) to find the GCF of 1200 and 1600.

Solution

a) The prime factorization of 12 and 16 are:

$$12 = 2 \times 2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$\text{GCF of } 12 \text{ and } 16 = 2 \times 2 = 4$$

b) We first note that

$$1200 = 12 \times 100 = 2 \times 2 \times 3 \times 100$$

$$1600 = 16 \times 100 = 2 \times 2 \times 2 \times 2 \times 100$$

and so the greatest common factor to 1200 and 1600 is given by

$$2 \times 2 \times 100 = 400$$

LCM:

the least common multiple, lowest common multiple, or smallest common multiple of two integers a and b , usually denoted by $\text{LCM}(a, b)$, is the smallest positive integer that is divisible by both a and b .

LCM by Listing out the Multiples

Find the LCM of 5 and 6

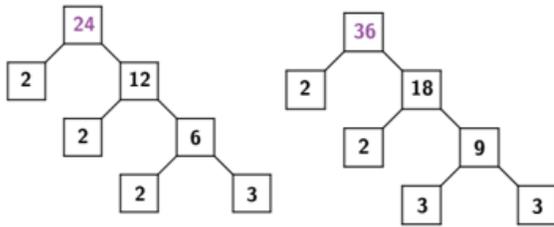
Multiples of 5: 5, 10, 15, 20, 25, **30**, 35, ...

Multiples of 6: 6, 12, 18, 24, **30**, 36, ...

Least Multiple common in both numbers is 30

LCM using Prime Factorization

Find the LCM of 24 and 36



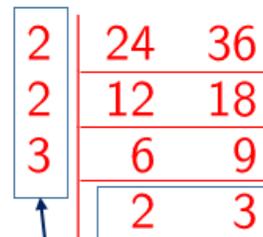
Box the common factors, list the numbers in each box once.

$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$

$$\text{LCM: } 2 \times 2 \times 3 \times 2 \times 3 = 72$$

LCM using Repeated Division

Find the LCM of 24 and 36



$$\text{LCM: } 2 \times 2 \times 3 \times 2 \times 3 = 72$$

Example:

Q1: What is the LCM of 4 and 6?

Multiples of 4 are:

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, ...

and the multiples of 6 are:

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, ...

Common multiples of 4 and 6 are simply the numbers that are in both lists:

12, 24, 36, 48, 60, 72,

So, from this list of the first few common multiples of the numbers 4 and 6, their least common multiple is 12

Two Methods to Find the LCM:

Method 1: May be used for small numbers:

Examples: Find lowest common multiple of the numbers 6 and 8.

List the first few multiples of both numbers and stop as soon as you find a common multiple.

1) Multiply 6 by 1, 2, 3,.. to obtain the first few multiples of 6:

6, 12, 18, 24, 30, 36 ...

2) Multiply 8 by 1, 2, 3,.. to obtain the first few multiples of 8:

8, 16, 24, 32, 40 ...

Examine the multiples: The lowest common multiple of 6 and 8 is 24.

Method 2: It uses prime factorization may be used for all numbers:

Examples: Find lowest common multiple (LCM) of the numbers 42 and 60.

step 1 - The prime factorization of 42 is: $42 = 2 \times 3 \times 7$

step 2 - The prime factorization of 60 is: $60 = 2 \times 2 \times 3 \times 5$

step 3 - The LCM is given by product of all prime number in the prime factorization with the highest power.

$$= 2 \times 2 \times 3 \times 7 \times 5 = 420$$

SOL QUESTIONS:

Q1: Find the lowest common multiple of 5 and 15.

Solution

The prime factorization of 5 and 15 are:

$$5 = 5$$

$$15 = 3 \times 5$$

The LCM is given by product of all prime number in the prime factorization with the highest power. Hence

$$\text{LCM of 5 and 15} = 5^1 \times 3^1 = 15$$

Q2: Find the lowest common multiple of 8, 12 and 18.

Solution

The prime factorization of 8, 12 and 18 are:

$$8 = 2 \times 2 \times 2 = 2^3$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

The LCM is given by product of all prime number in the prime factorization with the highest power.

$$\text{LCM of 8, 12 and 18} = 2^3 \times 3^2 = 72$$

Q3: Find the lowest common multiple of 70 and 90.

Solution

The prime factorization of 70 and 90 are:

$$70 = 2 \times 5 \times 7 = 2 \times 5 \times 7$$

$$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$$

The LCM is given by product of all prime number in the prime factorization with the highest power.

$$\text{LCM of 70 and 90} = 2 \times 5 \times 7 \times 3^2 = 630$$

Q4: What is the lowest common multiple of 180, 216 and 450?

The prime factorization of 180, 216 and 450:

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$$

$$450 = 2 \times 3 \times 3 \times 5 \times 5 = 2 \times 3^2 \times 5^2$$

The LCM is given by product of all prime number in the prime factorization with the highest power.

$$\text{LCM of } 180, 216 \text{ and } 450 = 2^3 \times 3^3 \times 5^2 = 5400$$