Q1: What is the Probability?

Probability mainly concerned with drawing inferences from uncertain situations.

e.g:- If we make a statement that we have fair chance to get first division in msc statistics degree.

Q2: How many Approaches of probability?

There are two approaches of probability.

1) Objective Approach
2) Subjective Approach

Q.3: Define objective approach of probability

Ans: Here we define the following definitions of probability

a) Classical definition of probability or Priori definition of probability

If a random experiment produces "n" equally likely and mutually exclusive outcomes out of which "m" outcomes are favorable to the event A, Then

$$P(A) = \frac{Number\,of\;\; favourable\;\; outcomes\,to\,A}{Toatal\;\; number\,of\;\; possible\;\; outcomes} = \frac{m}{n}$$

b) Relative frequency or A posterior definition of probability/Emperical definition

If a random experiment is repeated large number of times (say n) under identical conditions and if "m" outcomes are favourable to event A, then

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

This definition is also called empirical or statistical definition of probability

c) The axiomatic definition of probability

This definition is based on the following three axioms

i) For any event E_i; $0 \le P(E_i) \le 1$

ii) For sure event S; P(S) =1

iii) For “A and B” are mutually exclusive events

$P(A \cup B) = P(A) + P(B)$

Q4: Define Subjective approach of probability.

Ans: The probability based on personal feelings of a person is called subjective approach

Or

Arithmetic measure of uncertainty associated with a problem is called probability

Q5.: What is the range of probability?

Ans: The range of probability is from zero to one i.e. $0 \le P \le 1$

Q.6: When does probability becomes negative?

Ans: Probability never becomes negative. It always ranges from zero to one.

Q7:What is the variable?

A quantity or characteristics that varies , is known as variables.

e.g: Height , weight , age, IQ level.

Q8: What is Discrete Variable?

A variable which consisit of or takes whole number values is called discrete variables.

e.g 1) No of children 2) Rooms in a house 3) Runs scored by a player etc

Q9: What is continuous variable?

A variable which can assume all possible values on a continuous scale in a given interval is called a continuous variable.

e.g :

1) Height of the children 2) Weight of student 3) Temperature of the atmosphere

Q.10: Define random experiment with example.

An experiment in which outcomes vary from trial to trail is called random experiment.

a) Throwing a balanced die b) Tossing a coin c) Drawing a card from a will shuffled deck of 52 playing cards, etc.

Q11:What is the Random Variable?

A variable whose values are determined by the outcomes of a random experiment is called random variable.

Q.12: Write down the properties of random experiment.

Ans: There are three properties of random experiment are given below

a) The experiment can be repeated, practically or theoretically, any number of times.

b) The experiment always has two or more possible outcomes. An experiment that has only one possible outcome is not a random experiment.

c) The outcome of each repetition is unpredictable, i.e. it has some degree of uncertainty.

Q.13: Define Trial example.

Ans: Single performance of an experiment is called trial. For example tossing a coin is a trial and the occurrence of "H or T' its outcomes.

Q.14: Define outcomes with example.

Ans: The result of a trial is called an outcome. For example tossing a coin is a trial and the occurrence of "H or T' its outcomes. Where "H=Head and T=trial".

Q.15: Define Sample space with example.

Ans: All possible outcomes of a random experiment is called sample space. It is denoted by "S" For Example If we throw a single die then the sample space is $S = [1,2,3,4,5,6]$

Q.16: Define Sample points with example.

Ans: The units of sample space are called sample points. For example, if we toss two coins then sample space $S=[HH,HT,TH,TT]$ and HH,HT,TH,TT are sample points.

Q.17: Define event with example.

Ans: The subset of a sample space "S" is called an event. Events are usually denoted by Capital letters i.e. A, B, C, etc. For Example if we throw a single die then the sample space is $S=[1,2,3,4,5,6]$ where $A(even)=[,2,,4,,6]$ $B(odd)=[1,,3,,5]$ then "A and B" are events.

Q.18: Difference between simple event and compound event.

Ans: Simple event: An even that contains only one sample point of sample space is called simple event. . For example, if we toss two coins then sample space $S=[HH,HT,TH,TT]$ and $[HH]$ are the example of simple event.

Compound event: An even that contains at least two sample point of sample space is called compound event. . For example, if we toss two coins then sample space $S=[HH,HT,TH,TT]$ and $[HT,TH]$ are the example of compound event.

Q.19: Define sure event.

Ans: An event that consists of the sample space is called sure event. The probability of occurrence of sure event is always 1.

Q.20: Define impossible event.

Ans: An event which contains no unit of the sample space is called impossible event. The probability of occurrence of impossible event is always zero. Null set or empty set and empty set is also called impossible event because it contains no unit.

Q21: Define the term exhaustive events.

Ans: The events whose union makes the sample space are called exhaustive events. For example if a coin is tossed events $A=[H]$ and $B=[T]$ are exhaustive be cause their union generates the sample space of a coin i.e. $S=[H,T]$. It is also called collectively exhaustive events.

Q22: Define equally likely events with example.

Ans: Two events "A and B" are said to be equally likely if they have same chance of occurrence. i.e. $n(A)=n(B)$

e.g: When we toss a coin, the head is as likely to occur tail. Similarly, when we throw a die all six faces are equally likely.

Q23: Define not- equally likely events with example.

Ans: Two events "A and B" are said to be not- equally likely if they have not same chance of occurrence. i.e. $n(A)\neq n(B)$ when we throw a match-box, all six sides are not equally likely.

Q24: Define mutually exclusive events with example.

Ans: Two events "A and B" are said to be mutually exclusive or disjoint events if they have no sample points are common. For Example if we throw a single die then the sample space is $S=[1,2,3,4,5,6]$ where $A(even)=[,2,,4,,6]$ $B(odd)=[1,,3,,5]$ then "A and B" are mutually exclusive events because $A\cap B=\phi$

Q25: Define not-mutually exclusive events with example.

Ans: Two events "A and B" are said to be not- mutually exclusive or joint events if they have some sample points are common. For Example if we throw a single die then the sample space is $S=[1,2,3,4,5,6]$ where $A(even)=[,2,,4,,6]$ $B(face is\ prime)=[2,,3,,5]$ then "A and B" are not-mutually exclusive events because $A\cap B=[2]\neq\phi$

Q26: Define independent events with example.

Ans: Two events "A and B" are said to be independent if the occurrence of events "A" does not affect the occurrence of the event "B". The process of with replacement is attached with independent events.

Or

Two events "A and B"are said to be independent if and only if $P(A\cap B)=P(A).P(B)$

Or $P(A/B)=P(A)$ or $P(B/A)=P(B)$

Similarly for three independent vents “A, B and C” $P(A \cap B \cap C) = P(A)P(B)P(C)$

For example: In the experiment of tossing a coin three times the sample space $S = [HHH, HHT, HTH, HTT, THH, THT, TTH, TTT]$. Let “A= Event that outcomes has both head and tail= $A = [HHT, HTH, HTT, THH, THT, TTH]$ and “B= outcomes has at most one head= $B = [HTT, THT, TTH, TTT]$ $A \cap B = [HTH, HTT, THT]$

$P(A) = \frac{6}{8} = \frac{3}{4}$ $P(B) = \frac{4}{8} = \frac{1}{2}$

Q27: Define dependents events with example.

Ans: Two events “A and B” are said to be dependent if the occurrence of events “A” affect the occurrence of the event “B”. The process of without replacement is attached with dependent events.

Or

Two events “A and B”are said to be dependent if and only if $P(A \cap B) \neq P(A)P(B)$

Or $P(A/B) \neq P(A)$ or $P(B/A) \neq P(B)$

Or

Two events “A and B”are said to be dependent if $P(A/B) \neq P(A)$ or $P(B/A) \neq P(B)$

But they are $P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$ or $P(A \cap B) = P(B)P(A/B)$

Similarly for three events “A, B and C”

$P(A \cap B \cap C) = P(B)P(A/B)P(C/A \cap B)$

Or $P(A \cap B \cap C) = P(C)P(A/C)P(B/A \cap C)$

Or $P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$

Q28: Define conditional probability.

Ans: The probability associated with reduced sample space is called conditional probability. In symbol it is denoted by $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Or $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

Q29: If "A and B" are two independent events such that $P(A) = 0.2$ and $P(B) = 0.15$ then evaluate $P(A \cap B)$ and $P(A/B)$.

Ans: As we know that when two events are independent $P(A \cap B) = P(A)P(B)$

a) $P(A \cap B) = 0.2 \times 0.15 = 0.03$

b) $P(A/B) = P(A) = 0.2$ or $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.03}{0.15} = 0.2$

Q30: If $P(A) = 0.6$ and $P(B/A) = 0.4$ then find $P(A \cap B)$

Ans:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$0.4 = \frac{P(A \cap B)}{0.6}$$

$$P(A \cap B) = 0.24$$

Q31: Define the collectively exhaustive events?

Two events A & B are said to be collectively exhaustive when their union is 'S'

i-e AUB = S

e.g: Throwing a die

S = { 1,2,3,4,5,6}

A = Face is odd.

B = Face is even

A = {1,3,5}

B = {2,4,6}

AUB = {1,2,3,4,5,6} = S

So, Event A & B are collectively exhaustive events.

Sample space:-

P(AUB) = P(A) + P(B)

= P(S)

= 1

Define factorial with examples.

Ans: The product of first "n" natural numbers including "n" is called factorial

i.e. $n! = n \times (n-1) \times (n-2) \times ... \times 1$

For example: 5! =5*4*3*2*1=120   4! =4*3*2*1=24   2! =2*1=2   1! =1 0! =1

Define permutations with example.

Ans: Possible ways in which a finite number of units can be arranged in different sequence considering the order of their occurrence are called permutations. The number of "n" objects taking "r" at a time is denoted by ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

For example ${}^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5*4*3*2!}{2!} = 5*4*3 = 60$

Define Combination with example.

Ans: Possible ways in which a finite number of units can be arranged in different sequence ignoring the order of their occurrence are called combination. The number of "n" objects taking "r" at a time is denoted by ${}^{n}c_{r} = \frac{n!}{r!(n-r)!}$

For example ${}^{5}P_{3} = \frac{5!}{3!(5-3)!} = \frac{5*4*3!}{3!*2*1} = 5*4*3 = 10$

What is the relationship between ${}^{n}c_{r}$ and ${}^{n}P_{r}$?

Ans: we know that ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ and ${}^{n}c_{r} = \frac{n!}{r!(n-r)!}$

The relationship between these ${}^{n}c_{r}$ and ${}^{n}P_{r}$ are ${}^{n}c_{r} = \frac{{}^{n}P_{r}}{r!}$

How many permutations can be formed the word "Pakpattan"?

Ans: Total number of letters in the word "Pakpattan"=9=n

Number of P's=2=n_1

Number of A's=3=n_2

Number of T's=2=n_3

Number of K's=1=n_4

Number of N's=1=n_5

$$Possible\ \ ways = P = \frac{n!}{n_1!\times n_2 \times n_3 \times \ldots \times n_k} = \frac{9!}{2!\times 3!\times 2!\times 1!\times 1!} = \frac{9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{2\times 1\times 3\times 2\times 1\times 2\times 1\times 1\times 1} = 15120$$

Q32: State the addition theorem of probability for mutually exclusive events.

Ans: If "A and B" are two mutually exclusive events, then the probability that "A or B or both" occurs is given by: $P(A \cup B) = P(A) + P(B)$

Numerical Example:-

S = {1,2,3,4……….10} ∴ n(s) = n

A = {1,2,3} B = {5,6,7,8}

P(A) = $\frac{n(A)}{n(S)} = \frac{3}{10}$

P(B) = $\frac{n(B)}{n(S)} = \frac{4}{10}$

AUB = {1,2,3,5,6,7,8}

P(AUB) = $\frac{n(AUB)}{n(S)} = \frac{7}{10}$

P(AUB) = $\frac{3}{10} + \frac{4}{10}$

P(AUB) = P(A) + P(B)

Q33: State the addition theorem of probability for not-mutually exclusive events.

Ans: If “A and B” are two not-mutually exclusive events, then the probability that “A or B or both” occurs is given by: $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$

Numerical Example:-

S = {1,2,3,4………….10}

A = {2,4,6}   B = {2,3,4,5,7,8}

P (A) = 3/10   P (B) = 6/10

A∩B = {2,4}

$P(A \cap B) = \frac{2}{10}$

AUB = {2,3,4,5,6,7,8}

$P(AUB) = \frac{7}{10}$

$P(AUB) = \frac{3}{10} + \frac{6}{10} - \frac{2}{10}$

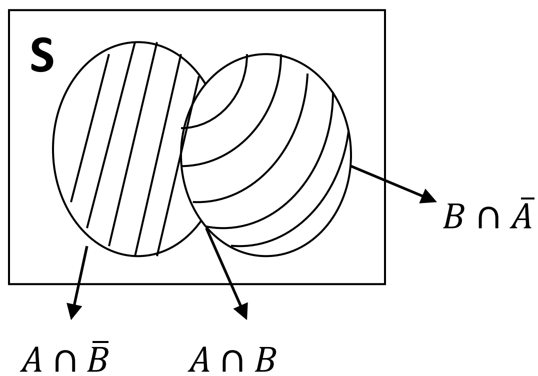
P (AUB) = P (A) + P (B) – P (A∩B)

Theorem:-

Additional law of probability for two not mutually exclusive events:-

Proof:-

Let A & B are two not mutually exclusive events then from venn diagram



$$A \cup B \text{ is shaded} = S$$

$A \cup B$ *may be written as*

$$A \cup B = (\, A \cap \bar{B}) \cup (A \cap B) \cup (B \cap \bar{A})$$

Applying probability on both side

$$P(A \cup B) = P(\, A \cap \bar{B}) + P(A \cap B) + P(B \cap \bar{A}) \ldots \ldots \ldots \ldots (1)$$

We know

$$A \cap \bar{B} = A - A \cap B$$

$$P(A \cap \overline{B)} = P(A) - P(A \cap B) \ldots \ldots \ldots (2)$$

&

$$B \cap \bar{A} = B - B \cap A$$

$$P(B \cap \overline{A)} = P(B) - P(B \cap A) \ldots \ldots \ldots \ldots .(3)$$

Putting the eq 2 & 3 in eq 1

$$P(A \cup B) = P(A) - P(\cancel{A \cap B}) + P(\cancel{A \cap B}) + P(B) - P(A \cap B)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This is called Additional law of probability for two not mutually exclusive events.

Theorem:-

Additional law of probability for three not mutually exclusive events.

Proof:-

If A , B & C are three not mutually exclusive events then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Let

$$A \cup B \cup C = A \cup (B \cup C)$$
$$= A \cup D \qquad \therefore D = B \cup C$$

Then

$$P(A \cup B \cup C) = P(A \cup D)$$
$$= P(A) + P(D) - P(A \cap D)$$

Interchange the value of D

$$= P(A) + P(B \cup C) - P[A \cap (B \cup C)]$$
$$= P(A) + P(B) + P(C) - P(B \cap C) - P[A \cap (B \cup C)] \ldots\ldots\ldots..(1)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$$
$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

Putting the result in eq 1

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Assignment:-

P(A) = 0.12 **P(B) = 0.89** **P(A∩B) = 0.07**

To find

1) P (AUB) =?
2) $P(AU\overline{B}\,)$ =?
3) $P(\bar{A}\;UB)$ =?
4) $P(\bar{A}\;U\;\bar{B})$ =?
5) $P(\bar{A}\;\cap B)$ =?
6) $P(A\cap\;\bar{B})$ =?
7) $P(\bar{A}\;\cap\;\bar{B})$ =?

Q 1: P(AUB) = P(A) + P(B) – P(A∩B)

0.12 + 0.89 – 0.07 = 0.94

Q 2: P(AU$\overline{\boldsymbol{B}}$) = P [A ∪ $(\boldsymbol{S}-\boldsymbol{B})$]

=P[(A∪ $S) - (A\cup B)$]

=P[S- $(A\cup B)$]

=P(S) – P(AUB)

=P(S) – [P(A) + P(B) – P(A∩ B)]

= 1 – [0.1 + 0.89 – 0.07] = 0.06

Q 3: Three coins are tossed. What is the probability that three tails results given that at least one tail.

Ans: Let the sample space "S" when three coins are tossed.

S = {HHH,HHT,HTH,THH,TTT,TTH,THT,HTT}

A be the event that contain at least one tail

A = {HHT,HTH,HTT,THH,TTH,THT,TTT}

P(A) = $\frac{n(A)}{n(S)}$ = 7/8

B be the event that three tails

B = {TTT}

P(B) = $\frac{n(B)}{n(S)}$ = 1/8

A∩ $B = \frac{n(A \cap B)}{n(S)}$ = 1/8

By conditionally probability

P(B/A) = $\frac{P(A \cap B}{P(A)}$ = $\frac{1/8}{7/8}$ = 1/7

Q 4: An integer is chosen at random from the first 200 positive integer what is the

Probability that integer is chosen divisible by 6 OR by 8?

All Possible sample

S= {1,2,3,4…………….200}

A denoted the event that first 200 positive integer is divisible by 6

n(A) = 200/6 = 33 points

P(A) = $\frac{n(A)}{n(S)}$ = 33/200

B denoted the event that first 200 positive integer divisible by 8.

n(B) = 200/8 = 25 points

P(B) = $\frac{n(B)}{n(S)}$ = 25/200

Events A & B are not mutually exclusive event as they have common the point divisible by 6 & 8 if and only if it is divisible by 24 then

n(A ∩ B) = 200/24 = 8

There are 8 figure that is divisible by 6 & 8 both then

P(A ∩ $B) = \frac{n(A \cap B)}{n(S)}$ = 8/200

P(AUB) = P(A) +P(B) – P(A ∩ B)

= 33/200 + 25/200 – 8/200

= $\frac{33+25-8}{200} = \frac{50}{200} = \frac{1}{4}$

Q 5: One integer is chosen at random from first 100 positive integers. What is the probability that the chosen number is divisible by 4 or 6 or 8.

Let the sample space

S = {1,2,3,4………..100}

A be the event divisible by 4

B be the event divisible by 6

C be the event divisible by 8

(A ∩ B) are the common event both in A & B.

n(A) = 100/4 = 25 n(B) = 100/6 = 16 n(C) = 100/8 = 12

n(A∩B) = 100/12 = 8 n(A∩C) = 100/8 = 12 n(B∩C) = 100/24 = 4

n(A∩B∩C) = 100/24 = 4

P(A) = $\frac{n(A)}{n(S)} = \frac{25}{100}$ P(B) = $\frac{n(B)}{n(S)} = \frac{16}{100}$ P(C) = $\frac{n(C)}{n(S)} = \frac{12}{100}$

P(A∩B) = $\frac{n(A \cap \mathrm{B})}{n(S)} = \frac{8}{100}$ P(A∩C) = $\frac{n(A \cap \mathrm{C})}{n(S)} = \frac{12}{100}$ P(B∩C) = $\frac{n(B \cap \mathrm{C})}{n(S)} = \frac{4}{100}$

P(A∩B∩C) = $\frac{n(A\cap B\cap C)}{n(S)} = \frac{4}{100}$

P(AUBUC) = P(A) + P(B) + P(C) – P(A∩B) – P(A∩C) – P(B∩C) + P(A∩B∩C)

$$= \frac{25}{100} + \frac{16}{100} + \frac{12}{100} - \frac{8}{100} - \frac{12}{100} - \frac{4}{100} + \frac{4}{100}$$ = 33/100 = 0.33

Q 6: A card is drawn at random from a deck of ordinary plying cards. What is the probability that is a diamonds, face cards & kings?

Let S be the sample space contain all 52 cards.

A be the event that contain Diamonds' cards,

B be the event that contain face cards,

C be the event that contain King cards,

S = {Ace 2,3,4…….52}

n(A) = 13 n(B) = 12 n(C) = 4

n(A∩B) = 3 n(A∩C) = 1 n(B∩C) = 4

n(A∩B∩C) = 1

P(A) = $\frac{n(A)}{n(S)} = \frac{13}{52}$ P(B) = $\frac{n(B)}{n(S)} = \frac{12}{52}$ P(C) = $\frac{n(C)}{n(S)} = \frac{4}{52}$

P(A∩B) = $\frac{n(A\cap B)}{n(S)} = \frac{3}{52}$ P(A∩C) = $\frac{n(A\cap C)}{n(S)} = \frac{1}{52}$ P(B∩C) = $\frac{n(B\cap C)}{n(S)} = \frac{4}{52}$

P(A∩B∩C) = $\frac{n(A\cap B\cap C)}{n(S)} = \frac{1}{52}$

P(AUBUC) = P(A) + P(B) + P(C) – P(A∩B) – P(A∩C) – P(B∩C) + P(A∩B∩C)

$$= \frac{13}{52} + \frac{12}{52} + \frac{4}{52} - \frac{3}{52} - \frac{1}{52} - \frac{4}{52} + \frac{1}{52}$$ = 22/52 = 0.4231

Q34: State the multiplication law of two independent events.

Ans: If "A and B" are two independent events then $P(A \cap B) = P(A)P(B)$

Q35: State the multiplication law of two dependent events.

Ans: If "A and B" are two dependent events if and only if

Then $P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$

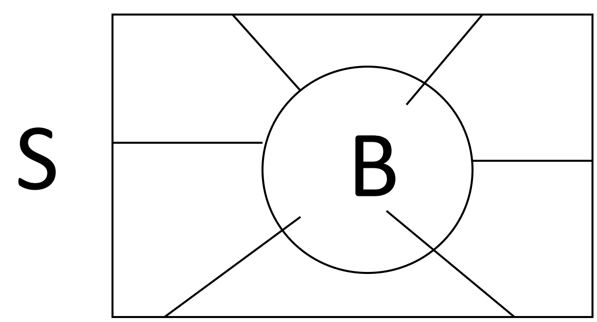
Bays' Theorem:-

Let A_1 , A_2 , A_3 are the mutually exclusive and collectively exhaustive events and B is the another event that occur if any one of the A_i occur.

$$P(B) = \sum_{i=1}^{n} P(Ai)P(B/Ai)$$

$$\boldsymbol{P\left(\frac{Ai}{B}\right) = \frac{P\,(Ai)P\left(\frac{B}{Ai}\right)}{\sum_{i=1}^{n} P(Ai)P\left(\frac{B}{Ai}\right)}}$$ for i= 1,2,3………n

Proof:-



$\bigcup_{i=1}^{n} Ai$ = S

A_1 U A_2 U A_3 U………U A_n = S

B is the subset of S

B = B∩S

B = B∩$\bigcup_{i=1}^{n} Ai$

B = B ∩ (A_1 U A_2 U A_3 U……… U A_n)

B = (B∩A_1) U (B∩A_2) U (B∩A_3)…………U (B∩A_n)

Applying probability on both sides

P(B) = P(B∩A_1) U P(B∩A_2) U P(B∩A_3)…………U P(B∩A_n)

Since Ai are the mutually exclusive.

So B∩Ai are the mutually exclusive.

P(B) = P(B∩A_1) + P(B∩A_2) + P(B∩A_3)…………+ P(B∩A_n)

By Multiplication Law of probability

P (B/Ai) = $\frac{P(A\cap B)}{P(Ai)}$

P(A∩B) = P(Ai) . P (B/Ai)

P(B) = $P(A_1).P(B/A_1)+ P(A_2).P(B/A_2)+ P(A_3).P(B/A_3)$……….. $P(A_n).P(B/A_n)$

P(B) = $\sum_{i=1}^{n} P(Ai).P\left(\frac{B}{Ai}\right)$

Conditional probability is

P(A∩B) = P(A). P(B/A) ……………….(1)

P(A∩B) = P(B). P(A/B) ……………….(2)

By comparing 1 & 2

P(B).P(A/B) = P(A).P(B/A)

P(A/B) = $\frac{P(A).P(\frac{B}{A})}{P(B)}$

Putting the value of P(B)

P(Ai/B) = $\frac{P(Ai).P(\frac{B}{Ai})}{\sum_{i=1}^{n} P(Ai).P(\frac{B}{Ai})}$

Hence proved

Random Variable:

A set of numerical values given to all possible outcomes of a random experiment

Is called random variable or chance variable or stochastic variable.

Example:

Throwing a die is a random experiment and the outcomes 1 , 2 , 3 , 4 , 5 ,6 are the values of a random variable.

Random variable is usually denoted by capitals letter X,Y,Z etc and values taken by these variables are denoted by small letters such as x ,y & z.

There are two types of random variable

1) Discrete variable
2) Continuous variable

Discrete random variable:

A random variable "X" is defined to be discrete random variable. If it can take values which are finite (X_1 , X_2 , X_3 …………X_n) such random variables are countable.

Example:

1) Number of heads obtained in a coin tossing experiment.
2) Number of defective items observed in a bundle.

Continuous random variable:

A random variable "X" is defined to be continuous random variable. If it can assume every possible value in an interval [a,b] where

$a<b$, a & b may be $-\infty$ & $+\infty$

Example:

1) The amount of rainfall in a certain city.

2) The speed of a car
3) The height of a person

Differentiate between discrete and continuous variable with example:

A variable which take all possible values in the given range is called Discrete variable, for example number of student in a class , number of children in a family etc . Discrete variable is a countable variable.

A variable which take selected values in the given interval is called continuous variable for example height, weight etc

Describe the discrete probability distribution and write down its properties:

Arrangement of all possible values of discrete variable "X" with their probabilities is called discrete probability distribution as shown below

X	X	X	.	.	.	.	X
$P(X_1)$	$P(X_2)$	$P(X_3)$	.	.	.	.	$P(X_n)$

Properties:

1) Each probability $P(X_i) \geq 0$
2) Sum of all probabilities is one $\sum P(Xi) = 1$

Example:

For the following discrete probability distribution find mean & variance.

X	P(X)	X P(X	$X^2P(X)$
1	1/4	1/4	1/4
2	1/4	2/4	4/4
3	1/4	3/4	9/4
4	1/4	4/4	16/4
	$\sum P(X) = \frac{4}{4} = 1$	$\sum XP(X)$ = 10/4 = 5/2 = 2.5	$\sum X^2P(X)$ = 30/4 = 15/2= 7.5

Mean = E(X) = $\sum XP(X)$ = 10/4 = 5/2 = 2.5

Variance = Va(X) = $\sum X^2P(X) - [\sum XP(X)]^2$ = $E(X)^2 - [E(X)]^2$

$= 30/4 - (10/4)^2$

$= 7.5 - (2.5)^2$

$= 7.5 - 6.25 = 1.25$

Standard Deviation = S.D = $\sqrt{Var\ (X)}$ = $\sqrt{1.25}$ = 0.5

Binomial random experiment:

There are many experiments which consist of repeated independent trail & each trail having only two possible complementary outcomes. e.g

1) Head & tail 2) Success & failure 3) right & wrong

4) alive & dead 5) good & defective etc

Probability of each outcome remains constant for all trials, then such trails are called Bernoulli trails and if experiments have n Bernoulli trails, it is called binomial experiment.

A binomial probability experiment possesses the following four properties:

1) Each trail may be classified either 'S' or 'F'
2) The probability of success remains constant for all trails
3) The successive trials are all independent
4) The experiment is repeated a fixed number of times

Binomial random variable:

In binomial probability experiment , X is called binomial random variable. Where

X = No of successes x = 0,1,2,3,………,n

Binomial Probability Distribution:

The probability distribution of binomial random variable (X) is called binomial probability distribution.

The random variable X takes anyone of (n + 1)th integer values 0,1,2,3,…..n.

Its p.df is P(x) = P(X=x) = $\binom{n}{x} q^{n-x} p^{x}$; x = 0,1,2,3,……..n Where q = 1-p ,

Binomial probability distribution has two parameters n & p and is denoted by

B(X ; n,p). It is approximate distribution when a random sample of size n is drawn with replacement from a finite population of size N or sampling is done from infinite population.

$$(q + p)^n = \sum_{x=0}^{n} \binom{n}{x} q^{n-x} p^{x}$$

$$= \sum_{x=0}^{n} b\ (x\ ; n\ , p) = 1$$

Binomial Frequency distribution:

If the binomial probability distribution is multiplied by N , the resulting distribution is called binomial probability distribution.

Mean = E(x) = $\mu = np$

Variance = var (x)= μ_2= δ^2 = npq where np > npq (always)

μ_3 = npq (1 – 2 p^2) = npq $(q - p)^2$

μ_4 = 3 n^2 p^2 q^2 + npq (1 – 6pq) = npq [1 + 3 (n - 2) pq]

Binomial Probability Distribution

Question: Find mean and variance of binomial distribution when "n=1"

Solution: Mean and variance of binomial distribution when "n" is one

X	P(x)	XP(X)	$X^2P(X)$
0 1	$\binom{1}{c\ 0}P^0q^1 = q$ $\binom{1}{c\ 1}P^1q^0 = P$	0 P	0 P
Total		$\sum XP(X) = p$	$\sum X^2P(X) = p$

$E(X) = \sum XP(X) = p$

$Var(X) = E(X^2) - (E(X))^2 = \sum X^2P(X) - (\sum XP(X))^2 = P - P^2 = P(1-P) = pq$

Question: Find mean and variance of binomial distribution when "n=2"

Solution:

Mean and variance of binomial distribution when "n" is two

X	P(x)	XP(X)	$X^2P(X)$
0 1 2	$\binom{2}{c\ 0}P^0q^2 = q^2$ $\binom{2}{c\ 1}P^1q^1 = 2pq$ $\binom{2}{c\ 2}p^2q^0 = p^2$	0 $2pq$ $2p^2$	0 $2pq$ $4p^2$
Total		$\sum XP(X) = 2pq + 2p^2$	$\sum X^2P(X) = 2pq + 4p^2$

$$E(X) = \sum XP(X) = 2pq + 2p^2 = 2p(q+p) = 2p((q+p) = 2p(1) = 2p$$

$$Var(X) = E(X^2) - (E(X))^2 = \sum X^2 P(X) - (\sum XP(X))^2 = 2pq + 4p^2 - (2p)^2 = 2pq + 4p^2 - 4p^2$$

$$Var(X) = 2pq + 4p^2 - 4p^2 = 2pq$$

$$S.D(X) = \sqrt{Var(X)} = \sqrt{2pq}$$

Short Questions

Q.1: Define Bernoulli trial?

Ans: A trial that gives only two possible outcomes is called Bernoulli trials.

Example:

There are many trials they have only two possible outcomes such as

i) Head and tail ii) Success and failure III) Alive and dead

iv) Right and wrong v) Good and defective

Q.2: What are the Properties of Bernoulli experiment?

Ans: There are four properties of Bernoulli experiment

i) Successive trials are Independent
ii) The probability of success remains same for all trials
iii) The experiment is repeated a single time
iv) Each trial classified into two categories such as success(S) or failure(F)

Q.3: Define binomial experiment.

Ans: An experiment consisting of "n" Bernoulli trials is known as binomial experiment.

Q.4: What is the Properties of Binomial experiment?

Ans: There are four properties of Binomial experiment

i) Successive trials are Independent
ii) The probability of success remains same for all trials
iii) The experiment is repeated a fixed number of times say "n"
iv) Each trial classified into two categories such as success(S) or failure(F)

Q.5: Explain briefly binomial random variable?

Ans: The random variable which denotes the number of successes of binomial experiment is called binomial random variable. It is discrete variable and assume values as X=0, 1, 2… n.

Q.6: Why is discrete probability function $b(x,n,p)=\binom{n}{\underset{x}{c}}P^x q^{n-x}$ called the binomial distribution?

Ans: $b(x,n,p)=\binom{n}{\underset{x}{c}}P^x q^{n-x}$ X=0, 1, 2… n. is called binomial distribution because its successive terms are equal to binomial expansion of $(q+p)^n$ and they give the probabilities of binomial random variable.

Q.7: What is binomial probability function or mass function?

Ans: The formula used to find the probabilities of binomial random variable "X" is called binomial probability function or binomial probability distribution. It is given by $b(x,n,p)=\binom{n}{\underset{x}{c}}P^x q^{n-x}$ X=0, 1, 2… n.

Where $q=1-p$ P= probability of success of single trial.

n= number independent of trials

Q.8: Define binomial probability distribution?

Ans: Arrangement of all possible values of binomial random variable of "X" with their probabilities is called binomial probability distribution and given as

X	P(x)

0	$\binom{n}{c_0} P^0 q^{n-0}$
1	$\binom{n}{c_1} P^1 q^{n-1}$
.	.
.	.
.	.
N	$\binom{n}{c_n} P^n q^{n-n}$

Q.9: Define binomial frequency distribution

Ans: If each probability of binomial distribution is multiplied by N then resulting distribution is called binomial frequency distribution as given below

X	NP(x)
0	$N\binom{n}{c_0} P^0 q^{n-0}$
1	$N\binom{n}{c_1} P^1 q^{n-1}$
.	.
.	.
.	.
N	$N\binom{n}{c_n} P^n q^{n-n}$

Q.10: Identify the parameters of binomial distribution $b(x, n, p)$?

Ans: “n” and “P” are the parameters of binomial probability distribution $b(x, n, p)$.

P= probability of success of single trial.   n= number independent of trials

Q.11: What are the parameters of binomial probability distribution?

Ans: “n” and “P” are the parameters of binomial probability distribution $b(x, n, p)$.

P= probability of success of single trial.   n= number independent of trials

Q.12: What is the range of binomial random variable "X"?

Ans: Binomial random variable "X" is a discrete variable and it ranges is from zero to "n"   i.e. X=0, 1, 2… n.

Q.13: Describe the properties of binomial distribution?

Ans: The properties of binomial distribution are given below

i) Total probability of binomial probability distribution is one.
ii) Mean of binomial distribution is "np"
iii) Variance of binomial distribution is "npq"
iv) Shape of binomial distribution $b(x, n, p)$ depends upon "n and p"

When $p = q = \frac{1}{2} or\ p = q = 0.5$ The distribution is always symmetrical

When $p \neq q$ The distribution is said to be skewed

When $p > \frac{1}{2} or\ p > 0.5$ The distribution is said to be negatively skewed

When $p < \frac{1}{2} or\ p < 0.5$ The distribution is said to be positively skewed

Q.14: How can the mean and variance of binomial probability distribution be calculated in terms of parameters?

Ans: Mean is calculated by multiplying by "n and p"   Mean=np

Variance is calculated by multiplying by "n, p and q"   Variance=npq

Q.15: Why in binomial distribution variance is smaller than mean give reason?

Ans: We know that in binomial distribution mean is "np" and variance is "np(1-p). To calculate variance mean is further multiplied by a number which is less than one hence Mean> Variance

Q.16: If the probability that it is a fine day is 0.7. Find the expected number of fine days in a week?

Ans: Given that P=0.7 and n=7

Mean=E(X) = np=7*0.7=4.9

Q.17: State the formula used to calculate binomial probabilities?

Ans: The formula used to find the probabilities of binomial random variable "X" is called binomial probability function or binomial probability distribution. It is given by $b(x,n,p)=\binom{n}{c\,x}P^{x}q^{n-x}$ X=0, 1, 2… n.

Where $q=1-p$ P= probability of success of single trial.

n= number independent of trials

Q.18: Which formulae's should be used to find the Mean, Variance, Standard deviation and Coefficient of variation of a binomial random variable?

Ans: To used the following formulae's to calculate Mean, Variance, Standard deviation and Coefficient of variation of a binomial random variable are given below

$$E(X) = Mean = np$$

$$Variance(X) = np(1-p) \quad or \;\; npq$$

$$S.D(X) = \sqrt{np(1-p)} \quad or \;\; \sqrt{npq}$$

$$Coefficien\,of \;\mathrm{var}\,iation = C.V = \frac{S.D(X)}{E(X)} \times 100$$