Ch.No.19 F-Distribution

Define F-Statistic and F distribution:
The ratio between two chi-square variable with their respective degree of freedom are called f-statistic

$U = \frac{(n_1 - 1)s_1^2}{\sigma_1^2}$ is χ^2 $-$distribution with $v_1 = n_1 - 1$ degree of freedom

$V = \frac{(n_2 - 1)s_2^2}{\sigma_2^2}$ is χ^2 $-$distribution with $v_2 = n_2 - 1$ degree of freedom

Then the ratio

$$F_c = \frac{U/v_1}{V/v_2} = \frac{(n_1-1)s_1^2/\sigma_1^2 \Big/ (n_1-1)}{(n_2-1)s_2^2/\sigma_2^2 \Big/ (n_2-1)} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{s_1^2\sigma_2^2}{s_2^2\sigma_1^2}$$

F-distribution:
Let "f" be a positive continuous random variable with interval $(0,\infty)$ is said to be "F" distribution and having its probability density function

$$f(F) = \frac{\left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}} F^{\frac{n_1}{2}-1}}{\beta(\frac{n_1}{2},\frac{n_2}{2})(1+\frac{n_1}{n_2}F)^{\frac{n_1+n_2}{2}}} \qquad 0 < F < \infty$$

It has two parameter n_1 and n_2 these are degree of freedom

Which test based on "F" distribution

i) Test the hypothesis about the equality or difference or ratio of two population variances or S.D $H_0 : \sigma_1^2 = \sigma_2^2$, when $n_1, n_2 < 120$.

ii) Test the homogeneity of "k" population means $H_0 : \mu_1 = \mu_2 = \mu_3 = ... = \mu_k$

iii) Test the homogeneity of "k" population variances $H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$. Cochran test

iv) Test the homogeneity of "k" population variances $H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$. Hartley test

v) Test the hypothesis about slope (regression coefficient) in SLRM $H_0 : \beta_1 = \beta_2$

vi) Test the hypothesis about slope (regression coefficient) in SLRM $H_0 : \beta_1 = \beta_2$ ANOVA table

vii) Test the hypothesis of overall test of significance regression in SLRM $H_0 : \beta = 0$ or $H_0 : \rho^2 = 0$

viii) Test the hypothesis about linearity of regression in SLRM.

ix) Test the hypothesis of overall test of significance regression in MLRM $H_0 : \rho^2 = 0$ or $H_0 : \beta_1 = \beta_2 = 0$

x) Test the hypothesis about population partial correlation coefficient $H_0 : \rho_{12.3...k} = 0$

x) Test the hypothesis about population multiple correlation coefficient $H_0 : \rho_{1.2.3...k} = 0$

Properties of "F" distribution

i) It is continuous probability distribution with interval $(0,\infty)$

ii) Total area under the curve is unity

iii) If n_1 *and* n_2 increases, the "F" distribution tends to normality

iv) If $n_2 \to \infty$ *and* $n_1 = 1$ then $\sqrt{F}$ tends to normality

v) If $n_2 \to \infty$ *Then* $n_1 F$ tends to χ^2 distribution with n_1 degree of freedom

vi) If $X \to F_{(n_1,n_2)}$ *Then* $\frac{1}{X} \to F_{(n_{21},n_1)}$

vii) I $X \to F_{(n_1,n_2)}$ then $Z = \dfrac{\frac{n_1}{n_2}X}{1+\frac{n_1}{n_2}X}$ follow beta distribution with parameter

$a = \dfrac{n_1}{2}$ *and* $b = \dfrac{n_2}{2}$

viii) The "F" distribution does not possess the m.g.f because some of moments are infinite

ix) "F" distribution is skewed to the right. But it can be shown that as n_1 and n_2 becomes large, the "F" distribution approaches the normal distribution.

x) The mean of "F" distribution is $\dfrac{n_2}{n_2-2}$ if $n_2 > 2$

xi) The variance of "F" distribution $\dfrac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)}$ if $n_2 > 4$

xii) The mode of "F" distribution $\dfrac{n_2(n_1-2)}{n_1(n_2+2)}$ if $n_1 > 2$ which is always less than one

xiii) The square of a "t" distribution with "n" degree of freedom has an "F" distribution with "1 and n" degree of freedom symbolically $\underset{n}{t^2} = \underset{(1,n)}{F}$

Assumption of "F" distribution

i) The samples are drawn randomly and independently

ii) The samples are selected from normally distributed. A slight departure from normality it is not serious.

Confidence interval for population variance ratio $\dfrac{\sigma_1^2}{\sigma_2^2}$

Let two independent random samples of size n_1 and n_2 are drawn from two normal populations with variances σ_1^2 *and* σ_2^2. If s_1^2 *and* s_2^2 are unbiased estimates of σ_1^2 *and* σ_2^2, then we know that

$U = \dfrac{(n_1-1)s_1^2}{\sigma_1^2}$ is χ^2 – distribution with $v_1 = n_1 - 1$ degree of freedom

$V = \dfrac{(n_2-1)s_2^2}{\sigma_2^2}$ is χ^2 – distribution with $v_2 = n_2 - 1$ degree of freedom

Then the ratio

$$F_c = \frac{U/v_1}{V/v_2} = \frac{(n_1-1)s_1^2/\sigma_1^2 \big/ (n_1-1)}{(n_2-1)s_2^2/\sigma_2^2 \big/ (n_2-1)} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{s_1^2\sigma_2^2}{s_2^2\sigma_1^2}$$

Then $100(1-\alpha)\%$ confidence interval for the ratio of two population variances in form of probability statement

$$P\left[F_{1-\frac{\alpha}{2}}(v_1,v_2) \le F_c \le F_{\frac{\alpha}{2}}(v_1,v_2)\right] = 1-\alpha$$

Then substituting the value of test statistic

$$P\left[F_{1-\frac{\alpha}{2}}(v_1,v_2) \le \frac{s_1^2\sigma_2^2}{s_2^2\sigma_1^2} \le F_{\frac{\alpha}{2}}(v_1,v_2)\right] = 1-\alpha$$

Multiplying each term inside the bracket of L.H.S by $\dfrac{s_2^2}{s_1^2}$

$$P\left[\frac{s_2^2}{s_1^2}F_{1-\frac{\alpha}{2}}(v_1,v_2) \le \frac{s_2^2}{s_1^2}\frac{s_1^2\sigma_2^2}{s_2^2\sigma_1^2} \le \frac{s_2^2}{s_1^2}F_{\frac{\alpha}{2}}(v_1,v_2)\right] = 1-\alpha$$

$$P\left[\frac{s_2^2}{s_1^2}F_{1-\frac{\alpha}{2}}(v_1,v_2) \le \frac{\sigma_2^2}{\sigma_1^2} \le \frac{s_2^2}{s_1^2}F_{\frac{\alpha}{2}}(v_1,v_2)\right] = 1-\alpha$$

Inverting each term then sign of inequality will be change

$$P\left[\frac{1}{\frac{s_2^2}{s_1^2}F_{1-\frac{\alpha}{2}}(v_1,v_2)} \geq \frac{1}{\frac{\sigma_2^2}{\sigma_1^2}} \geq \frac{1}{\frac{s_2^2}{s_1^2}F_{\frac{\alpha}{2}}(v_1,v_2)}\right]=1-\alpha$$

$$P\left[\frac{s_1^2}{s_2^2}\frac{1}{F_{1-\frac{\alpha}{2}}(v_1,v_2)} \geq \frac{\sigma_1^2}{\sigma_2^2} \geq \frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)}\right]=1-\alpha$$

By the property $\frac{1}{F_{1-\frac{\alpha}{2}}(v_1,v_2)} = F_{\frac{\alpha}{2}}(v_2,v_1)$

$$P\left[\frac{s_1^2}{s_2^2}F_{\frac{\alpha}{2}}(v_2,v_1) \geq \frac{\sigma_1^2}{\sigma_2^2} \geq \frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)}\right]=1-\alpha$$

Or

$$P\left[\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2}F_{\frac{\alpha}{2}}(v_2,v_1)\right]=1-\alpha$$

Hence the required result

We can also find confidence interval for the ratio of population standard deviation $\frac{\sigma_1}{\sigma_2}$

By taking the square root of end points of the confidence limits

Example.19.1: Given two random samples of size $n_1=12$ and $n_2=10$ from two independent normal populations, with $s_1=2.3$ and $s_2=1.5$, find 90% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$ and $\frac{\sigma_1}{\sigma_2}$.

Solution:

The $100(1-\alpha)\%$ Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2}F_{\frac{\alpha}{2}}(v_2,v_1)$$

Given that $n_1=12$ and $n_2=10$, $s_1^2=5.29$ and $s_2^2=2.25$ $\quad 1-\alpha=0.90 \Rightarrow \alpha=0.10$

$v_1=n_1-1=11$ *and* $v_2=n_2-1=9$ $F_{\frac{\alpha}{2}}(v_1,v_2)=F_{0.05}(11,9)=3.10$

and $F_{\frac{\alpha}{2}}(v_2,v_1)=F_{0.05}(9,11)=2.90$

Hence 90% Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2}F_{\frac{\alpha}{2}}(v_2,v_1)$$

$$\frac{5.29}{2.25}\left(\frac{1}{3.10}\right) \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{5.29}{2.25}(2.90)$$

$$0.76 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 6.81$$

Hence the required 90% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is “0.76 to 6.81”

Now confidence interval for $\frac{\sigma_1}{\sigma_2}$ for this we take the square root of the end limits

Hence the required 90% confidence interval for $\frac{\sigma_1}{\sigma_2}$ is “0.87 to 2.61”

Q.4 (b): Given $n_1=n_2=16$, $s_1^2=50$ and $s_2^2=16$, construct a 90% confidence interval for δ_1^2 / δ_2^2.

Solution:

The $100(1-\alpha)\%$ Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} F_{\frac{\alpha}{2}}(v_2,v_1)$$

Given that $n_1=n_2=16$, $s_1^2=50$ and $s_2^2=16$ $\quad 1-\alpha=0.90 \Rightarrow \alpha=0.10$

$v_1=n_1-1=15$ *and* $v_2=n_2-1=15$ $F_{\frac{\alpha}{2}}(v_1,v_2)=F_{0.05}(15,15)=2.40$

and $F_{\frac{\alpha}{2}}(v_2,v_1)=F_{0.05}(15,15)=2.40$

Hence 90% Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} F_{\frac{\alpha}{2}}(v_2,v_1)$$

$$\frac{50}{16}\frac{1}{2.4} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{50}{16}2.40$$

$$1.30 \le \frac{\sigma_1^2}{\sigma_2^2} \le 7.5$$

Hence the required 90% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is "1.30 to 7.5"

Q.4©: Given $n_1=41, n_2=13,\ s_1^2=15.6$ and $s_2^2=6.3$, Construct a 98 per cent confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$

Solution:

The $100(1-\alpha)\%$ Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} F_{\frac{\alpha}{2}}(v_2,v_1)$$

Given that $n_1=41, n_2=13,\ s_1^2=15.6$ and $s_2^2=6.3$ $\quad 1-\alpha=0.98 \Rightarrow \alpha=0.02$

$v_1=n_1-1=40$ *and* $v_2=n_2-1=12$ $F_{\frac{\alpha}{2}}(v_1,v_2)=F_{0.01}(40,12)=3.62$

and $F_{\frac{\alpha}{2}}(v_2,v_1)=F_{0.01}(12,40)=2.66$

Hence 90% Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} F_{\frac{\alpha}{2}}(v_2,v_1)$$

$$\frac{15.6}{6.3}\frac{1}{3.62} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{15.6}{6.3}2.66$$

$$0.68 \le \frac{\sigma_1^2}{\sigma_2^2} \le 6.59$$

Hence the required 98% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is "0.68 to 6.59"

Q.5 (b): Given two random samples of size $n_1=9$ and $n_2=16$ from two independent normal population ,with $s_1=6$ and $s_2=5$ find 98 % confidence intervals for δ_1^2 / δ_2^2 and δ_1 / δ_2.

Solution:

The $100(1-\alpha)\%$ Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2}F_{\frac{\alpha}{2}}(v_2,v_1)$$

Given that $n_1 = 9, n_2 = 16,\ s_1^2 = 36$ and $s_2^2 = 25$ $\quad 1-\alpha = 0.98 \Rightarrow \alpha = 0.02$

$v_1 = n_1 - 1 = 8$ *and* $v_2 = n_2 - 1 = 15$ $F_{\frac{\alpha}{2}}(v_1,v_2) = F_{0.01}(8,15) = 4.0$

and $F_{\frac{\alpha}{2}}(v_2,v_1) = F_{0.01}(15,8) = 5.52$

Hence 90% Confidence interval for the ratio of population variances $\frac{\sigma_1^2}{\sigma_2^2}$

$$\frac{s_1^2}{s_2^2}\frac{1}{F_{\frac{\alpha}{2}}(v_1,v_2)} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2}F_{\frac{\alpha}{2}}(v_2,v_1)$$

$$\frac{36}{25}\frac{1}{4.0} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{36}{25}5.52$$

$$0.36 \le \frac{\sigma_1^2}{\sigma_2^2} \le 7.95$$

Hence the required 98% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$ is “0.36 to 7.95”

Now confidence interval for $\frac{\sigma_1}{\sigma_2}$ for this we take the square root of the end limits

Hence the required 98% confidence interval for $\frac{\sigma_1}{\sigma_2}$ is “0.6 to 2.82”

Q.5©: A random sample of 10 salt-water fish had a variance, s_1^2 in girth of 7.2 (inches)^2 ,while a random sample of 8 fresh-water fish had a variance , s_2^2 in girth of 3.6 (inches)^2 .Find a 90 per cent confidence interval for the ratio between the two variance $\frac{\sigma_2^2}{\sigma_1^2}$.Assume normal Populations.

Solution:

The $100(1-\alpha)\%$ Confidence interval for the ratio of population variances $\frac{\sigma_2^2}{\sigma_1^2}$

$$\frac{s_2^2}{s_1^2}\frac{1}{F_{\frac{\alpha}{2}}(v_2,v_1)} \le \frac{\sigma_2^2}{\sigma_1^2} \le \frac{s_2^2}{s_1^2}F_{\frac{\alpha}{2}}(v_1,v_2)$$

Given that $n_1 = 10, n_2 = 8,\ s_1^2 = 7.2$ and $s_2^2 = 3.6$ $\quad 1-\alpha = 0.90 \Rightarrow \alpha = 0.10$

$v_1 = n_1 - 1 = 9$ *and* $v_2 = n_2 - 1 = 7$ $F_{\frac{\alpha}{2}}(v_1,v_2) = F_{0.01}(9,7) = 3.68$

and $F_{\frac{\alpha}{2}}(v_2,v_1) = F_{0.01}(7,9) = 3.29$

Hence 90% Confidence interval for the ratio of population variances $\frac{\sigma_2^2}{\sigma_1^2}$

$$\frac{s_2^2}{s_1^2}\frac{1}{F_{\frac{\alpha}{2}}(v_2,v_1)} \le \frac{\sigma_2^2}{\sigma_1^2} \le \frac{s_2^2}{s_1^2}F_{\frac{\alpha}{2}}(v_1,v_2)$$

$$\frac{3.6}{7.2}\frac{1}{3.29} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{3.6}{7.2}3.68 \qquad 0.15 \le \frac{\sigma_2^2}{\sigma_1^2} \le 1.84$$

Hence the required 90% confidence interval for $\frac{\sigma_2^2}{\sigma_1^2}$ is “0.15 to 3.68”

Q.6 (a): Describe how you would test the equality of two variance.
Solution:
Let two independent random samples of size n_1 and n_2 are drawn from two normal populations with variances σ_1^2 *and* σ_2^2. If s_1^2 *and* s_2^2 are unbiased estimates of σ_1^2 *and* σ_2^2, then we know that

$U = \frac{(n_1-1)s_1^2}{\sigma_1^2}$ is χ^2 – distribution with $v_1 = n_1 - 1$ degree of freedom

$V = \frac{(n_2-1)s_2^2}{\sigma_2^2}$ is χ^2 – distribution with $v_2 = n_2 - 1$ degree of freedom

Then the ratio

$$F_c = \frac{U/v_1}{V/v_2} = \frac{(n_1-1)s_1^2/\sigma_1^2 \Big/ (n_1-1)}{(n_2-1)s_2^2/\sigma_2^2 \Big/ (n_2-1)} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{s_1^2\sigma_2^2}{s_2^2\sigma_1^2}$$

Assuming that the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ then test statistic becomes

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

which is the required test-statistic with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom
Procedure
i) We state our null and alternative hypothesis

a) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$ b) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \geq 1$ c) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \leq 1$

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$ $H_0 : \frac{\sigma_1^2}{\sigma_2^2} < 1$ $H_0 : \frac{\sigma_1^2}{\sigma_2^2} > 1$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.
iii) Level of significance
$\alpha = Commonly\, we\, used$ 1% *or* 5%
iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom
v) Critical region
It is naturally based on alternative hypothesis

a) When $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$ then we used two sided test

$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2)$ if $s_1^2 > s_2^2$ and we use $F_c = \frac{s_1^2}{s_2^2}$

$F_c \geq F_{\frac{\alpha}{2}}(v_2, v_1)$ if $s_2^2 > s_1^2$ and we use $F_c = \frac{s_2^2}{s_1^2}$

Or
If one wishes to use a two sided test, then critical region will be

$$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \text{ and } F_c \leq F_{1-\frac{\alpha}{2}}(v_1, v_2) = \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)}$$

b) When $H_0 : \frac{\sigma_1^2}{\sigma_2^2} < 1$ then we used one sided test

Change the rules of two samples and test-Statistic $F_c = \frac{s_2^2}{s_1^2}$ critical region will be

$F_c > F_\alpha(v_2, v_1)$ if $s_2^2 > s_1^2$ and we use $F_c = \frac{s_2^2}{s_1^2}$

c) When $H_0 : \frac{\sigma_1^2}{\sigma_2^2} > 1$ then we used one sided test

$F_c > F_\alpha(v_1, v_2)$ if $s_1^2 > s_2^2$ and we use $F_c = \frac{s_1^2}{s_2^2}$

vi) Calculation

In this setup we calculate the value of test-statistic on the basis of sample information

vii) Conclusion:

If our calculated value does not fall's in critical region, then we accept H_0.

Example.19.2: Given two random samples of size $n_1 = 12$ and $n_2 = 10$ from two independent normal populations, with $s_1 = 2.3$ and $s_2 = 1.5$, test at 0.05 level of significance , the hypothesis $H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$ against the alternative $H_1 : \frac{\sigma_1^2}{\sigma_2^2} > 1$

Solution:

i) We state our null and alternative hypothesis

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_1 : \frac{\sigma_1^2}{\sigma_2^2} > 1$$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$F_c > F_\alpha(v_1, v_2) > F_{0.05}(11,9) > 3.10$

vi) Calculation

$s_1^2 = 5.29$ And $s_2^2 = 2.25$

$$F_c = \frac{s_1^2}{s_2^2} = \frac{5.29}{2.25} = 2.33$$

vii) Conclusion:

Since our calculated value does not fall's in critical region. So, we accept H_0 at 5% level of significance and we conclude that variances are equal.

Example.19.3: Two random samples drawn from two normal populations are

Sample I: 20, 16, 26,27,23,22,18,24,25 and 19

Sample II: 27,33,42,35,32,34,38,28,41,43,30 and 37 obtain the estimate of variances of the population and test whether the two population have the same variance.

Solution:

i) We state our null and alternative hypothesis

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$ Or $H_0 : \sigma_1^2 \neq \sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \geq F_{0.025}(9,11) \geq 3.59 \; and \; F_c \geq \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \leq \frac{1}{F_{0.025}(11,9)} \leq \frac{1}{3.92} \leq 0.26$

vi) Calculation

X_1	$\left(X_1 - \bar{X}_1\right)^2$	X_2	$\left(X_2 - \bar{X}_2\right)^2$
20		27	
16		33	
26		42	
27		35	
23		32	
22		34	
18		38	
24		28	
25		41	
19		43	
		30	
		37	

$s_1^2 = \frac{\sum(x_1 - \bar{x}_1)^2}{n_1 - 1} = 13.33$ And $s_1^2 = \frac{\sum(x_2 - \bar{x}_2)^2}{n_2 - 1} = 28.55$

$F_c = \frac{s_1^2}{s_2^2} = \frac{13.33}{28.55} = 0.47$

vii) Conclusion:

Since our calculated value does not fall's in critical region. So, we reject H_0 at 5% level of significance and we conclude that variances are equal.

Therefore variance is not equal, so we do not use t-test to test the equality of means. The assumption of equal variance is necessary to test $H_0 : \mu_1 = \mu_2$

Example: 19.4: In an experiment on reaction times in seconds of two individual "A and B" measured under identical conditions, the following results were obtained

A	0.41,0.38,0.37,0.42,0.35,0.38
B	0.32,0.36,0.38,0.33,0.38

a) Test the hypothesis at 0.05 level of significance test $H_0 : \sigma_A^2 = \sigma_B^2$ against $H_A : \sigma_A^2 \neq \sigma_B^2$

b) If $H_0 : \sigma_A^2 = \sigma_B^2$ is accepted in part (a), then test the hypothesis at 0.05 level of significance that $H_0 : \mu_A = \mu_B$ against $H_1 : \mu_A \neq \mu_B$

Solution:

i) We state our null and alternative hypothesis

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$ Or $H_0 : \sigma_1^2 \neq \sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance $\sigma_1^2 \; and \; \sigma_2^2$ are unbiased estimators of $s_1^2 \; and \; s_2^2$ when $n_1 \; and \; n_2 < 120$.

iii) Level of significance

$\alpha = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \geq F_{0.025}(5,4) \geq 9.36 \; and \; F_c \geq \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \leq \frac{1}{F_{0.025}(4,5)} \leq \frac{1}{7.39} \leq 0.14$$

vi) Calculation

X_1	$\left(X_1 - \overline{X}_1\right)^2$	X_2	$\left(X_2 - \overline{X}_2\right)^2$
0.41 0.38 0.37 0.42 0.35 0.38		0.32 0.36 0.38 0.33 0.38	

$$s_A^2 = \frac{\sum(x_1 - \bar{x}_1)^2}{n_1 - 1} = 0.0007 \quad \text{And} \quad s_B^2 = \frac{\sum(x_2 - \bar{x}_2)^2}{n_2 - 1} = 0.0008$$

$$F_c = \frac{s_2^2}{s_1^2} = \frac{0.0007}{0.0008} = 0.875$$

vii) Conclusion:

Since our calculated value does not fall's in critical region. So, we accept H_0 at 5% level of significance and we conclude that variances are equal.

Therefore variance is not equal, so we do not use t-test to test the equality of means. The assumption of equal variance is necessary to test $H_0 : \mu_1 = \mu_2$

b) Solution:

i) We set up our null and alternative hypothesis

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

ii) Assumption: The two samples of sizes $n_1 \; and \; n_2$ are randomly and independently drawn from two normal populations with population mean $\mu_1 \; and \; \mu_2$ when population variances are $\sigma_1^2 \; and \, \sigma_2^2$.unknown but $\sigma_1^2 = \sigma_2^2$ and sample sizes are small.

iii) Level of significance

$\alpha = 5\% = 0.05$

iv) Test-statistic

$$t_c = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

If H_0 is true; it hast-distribution with $\left(V = n_1 + n_2 - 2\right)$ degree of freedom

v) Critical region

$|t_c| \geq t_{\frac{\alpha}{2}}(v) \geq t_{0.025}(9) \geq 2.26$ $\quad v = 6 + 5 - 2 = 9$

vi) Calculation

$\overline{X}_A = 0.385$ and $\overline{X}_B = 0.354$

$$S_p = \sqrt{\frac{\sum\left(X_1 - \overline{X}_1\right)^2 + \sum\left(X_1 - \overline{X}_1\right)^2}{n_1 + n_2 - 2}} = 0.027$$

$$t_c = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(0.385 - 0.354) - (0)}{0.027\sqrt{\frac{1}{6} + \frac{1}{5}}} = 1.90$$

vii) Conclusion

Since our calculated value does not fall's in critical region so we accept H_0. And we conclude that the $H_0 : \mu_A = \mu_B$ at 5% level of significance.

Q.6 (b): Given the following information, what is your conclusion in testing each of the indicated null hypotheses?

	N1	n2	s_1^2	s_2^2	α	H0	H1
(i)	16	16	50	16	0.05	$\delta_1^2 / \delta_2^2 = 1$	$\delta_1^2 / \delta_2^2 > 1$
(ii)	13	41	6.3	15.6	0.01	$\delta_1^2 / \delta_2^2 = 1$	$\delta_1^2 / \delta_2^2 < 1$
(iii)	60	120	8.0	17.0	0.02	$\delta_1^2 / \delta_2^2 = 1$	$\delta_1^2 / \delta_2^2 \neq 1$

Solution:

i)

i) We state our null and alternative hypothesis

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_1 : \frac{\sigma_1^2}{\sigma_2^2} > 1$$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 5\% = 0.05$

iv) Test-Statistic

$$F_c = \frac{s_1^2}{s_2^2} \qquad \text{Therefore } s_1^2 > s_2^2$$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$$F_c > F_\alpha(v_1, v_2) \qquad \text{if } s_1^2 > s_2^2 \text{ and we use } F_c = \frac{s_1^2}{s_2^2}$$

$F_c > F_{0.05}(15{,}15)$

$F_c > 2.54$

vi) Calculation

	N1	n2	s_1^2	s_2^2	α	H0	H1
(i)	16	16	50	16	0.05	$\delta_1^2 / \delta_2^2 = 1$	$\delta_1^2 / \delta_2^2 > 1$

$$F_c = \frac{s_1^2}{s_2^2} = \frac{50}{16} = 3.12$$

vii) Conclusion:

Since our calculated value fall in critical region, so we reject H_0 at 5% level of significance.

ii) Solution

i) We state our null and alternative hypothesis

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_1 : \frac{\sigma_1^2}{\sigma_2^2} < 1$$

ii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 0.01 = 1\%$

iv) Test-Statistic

$$F_c = \frac{s_1^2}{s_2^2} \qquad \text{Therefore } s_1^2 > s_2^2$$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

When $H_0 : \frac{\sigma_1^2}{\sigma_2^2} < 1$ then we used one sided test

Change the rules of two samples and test-Statistic $F_c = \frac{s_2^2}{s_1^2}$ critical region will be

$F_c > F_\alpha(v_2, v_1)$ if $s_2^2 > s_1^2$ and we use $F_c = \frac{s_2^2}{s_1^2}$

$F_c > F_{0.01}(40{,}12) > 2.66$

vi) Calculation

	N_1	n_2	s_1^2	s_2^2	α	H_0	H_1
(ii)	13	41	6.3	15.6	0.01	$\delta_1^2 / \delta_2^2 = 1$	$\delta_1^2 / \delta_2^2 < 1$

$F_c = \frac{s_2^2}{s_1^2} = \frac{15.6}{6.3} = 2.48$

vii) Conclusion:

Since our calculated value does not fall's in critical region, so we accept H_0.

iii) Solution

i) We state our null and alternative hypothesis

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$

$H_1 : \frac{\sigma_1^2}{\sigma_2^2} < 1$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 0.02$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$F_c \geq F_{\frac{\alpha}{2}}(v_1 = 59, v_2 = 119) \geq 1.38$ if $s_1^2 > s_2^2$ and we use $F_c = \frac{s_1^2}{s_2^2}$

$F_c \geq F_{\frac{\alpha}{2}}(v_2 = 119, v_1 = 59) \geq$ if $s_2^2 > s_1^2$ and we use $F_c = \frac{s_2^2}{s_1^2}$

Or

vi) Calculation

	N_1	n_2	s_1^2	s_2^2	α	H_0	H_1
(iii)	60	120	8.0	17.0	0.02	$\delta_1^2 / \delta_2^2 = 1$	$\delta_1^2 / \delta_2^2 \neq 1$

$F_c = \frac{s_1^2}{s_2^2} = \frac{8}{17} = 0.47$

vii) Conclusion:

Since our calculated value does not fall's in critical region, so we accept H_0 at 2% level of significance.

Q.7: (a) Two independent random samples of size n_1=10 and n_2=7 were observed to have sample variance of s_1^2=16 and s_2^2=3. Using a 10% level of significance, test H_0: $\delta_1^2 = \delta_2^2$ against . H_1: $\delta_1^2 \neq \delta_2^2$ .Then using a 5% level of significance, test H_0: $\delta_1^2 = \delta_2^2$ and H_1: $\delta_1^2 > \delta_2^2$ and H_0: $\delta_1^2 < \delta_2^2$.

Solution:

i) We state our null and alternative hypothesis

a) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$ b) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \le 1$ c) $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \ge 1$

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} \ne 1$ $H_0 : \frac{\sigma_1^2}{\sigma_2^2} > 1$ $H_0 : \frac{\sigma_1^2}{\sigma_2^2} < 1$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

a) $\alpha = 0.10$ b) $\alpha = 0.05$ c) $\alpha = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

a) $F_c \ge F_{\frac{\alpha}{2}}(v_1, v_2) \ge F_{0.05}(9,6) \ge 4.11$ *and* $F_c \ge \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \le \frac{1}{F_{0.05}(6,9)} \le \frac{1}{3.37} \le 0.297$

b) When $H_0 : \frac{\sigma_1^2}{\sigma_2^2} > 1$ then we used one sided test

$F_c > F_\alpha(v_1, v_2) > F_{0.05}(9,6) > 4.10$

c) When $H_0 : \frac{\sigma_1^2}{\sigma_2^2} < 1$ then we used one sided test

Change the rules of two samples and test-Statistic $F_c = \frac{s_2^2}{s_1^2}$ critical region will be

$F_c > F_\alpha(v_2, v_1) > F_\alpha(6,9) > 3.37$

vi) Calculation

a) $F_c = \frac{s_1^2}{s_2^2} = \frac{16}{3} = 5.33$

b) $F_c = \frac{s_1^2}{s_2^2} = \frac{16}{3} = 5.33$

c) $F_c = \frac{s_2^2}{s_1^2} = \frac{3}{16} = 0.18$

vii) Conclusion:

Since our calculated value fall in critical region, so we reject H_0 in first two conditions. And in 3rd condition our calculated value does not fall's in critical region, so we accept H_0 at the given level of significance.

Q.7 (b): Two samples are randomly selected from two classes of students who have been taught by different methods. An examination is given and the results are shown as follows:

	Class 1	**Class 2**
Sample size	8	10
Means:	95	97
Unbiased sample variance	47	80

Test whether the two different methods of teaching are equally variable.

Solution:

i) We state our null and alternative hypothesis

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \ne \sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 5\% = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \geq F_{0.025}(7,9) \geq 4.0$ *and* $F_c \geq \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \leq \frac{1}{F_{0.025}(9,7)} \leq \frac{1}{4.84} \leq 0.21$

vi) Calculation

$F_c = \frac{s_1^2}{s_2^2} = \frac{80}{47} = 1.702$

vii) Conclusion:

Since our calculated value does not fall's in critical region, so we accept H_0 and we conclude that the two methods of teaching are equally variable.

Q.8: A standardized placement test in mathematics was given to 25 boys and 16 girls. The boys made an average grade of 82 with a standard deviation of 8, while the made an average grade of 78 with a standard deviation of 7. Test the hypothesis that $H_{0:}\ \delta_1^2 = \delta_2^2$ against the alternative hypothesis $H_{1:}\ \delta_1^2 \neq \delta_2^2$, where δ_1^2 and δ_2^2 the variance of the population of grades for all boys and girls respectively. Use a 0.02 level of significance.

Solution:

i) We state our null and alternative hypothesis

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 2\% = 0.02$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \geq F_{0.025}(9,11) \geq 3.59$ *and* $F_c \geq \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \leq \frac{1}{F_{0.025}(11,9)} \leq \frac{1}{3.92} \leq 0.26$

vi) Calculation

$F_c = \frac{s_1^2}{s_2^2} = \frac{64}{49} = 1.31$

vii) Conclusion:

Since our calculated value does not fall's in critical region, so we accept H_0 at 2% level of significance and we conclude that variances are equal.

Q.9: Independent random samples were selected from each of two normally distribution population, n_1=6 from population 1 and n_2=5 from population 2. The data are show below:

Sample 1: 3.1, 4.4, 1.2, 1.7, 0.7, 3.4

Sample 2: 2.3, 1.4, 3.7, 8.9, 5.5

Do these data provide sufficient evidence to indicate a difference between the population variance? Use α=0.05

Solution:

i) We state our null and alternative hypothesis

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 and σ_2^2 are unbiased estimators of s_1^2 and s_2^2 when n_1 and $n_2 < 120$.

iii) Level of significance

$\alpha = 5\% = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \geq F_{0.025}(4,5) \geq 7.39 \; and \; F_c \geq \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \leq \frac{1}{F_{0.025}(5,4)} \leq \frac{1}{9.36} \leq 0.11$$

vi) Calculation

X_1	$\left(X_1 - \bar{X}_1\right)^2$	X_2	$\left(X_2 - \bar{X}_2\right)^2$
3.1 4.4 1.2 1.7 0.7 3.4		2.3 1.4 3.7 8.9 5.5	

$s_1^2 = \frac{\sum\left(x_1 - \bar{x}_1\right)^2}{n_1 - 1} = 2.062$ And $s_2^2 = \frac{\sum\left(x_2 - \bar{x}_2\right)^2}{n_2 - 1} = 8.84$

$F_c = \frac{s_2^2}{s_1^2} = \frac{8.84}{2.062} = 4.29$

vii) Conclusion:

Since our calculated value does not fall's in critical region, so we accept H_0 at 5% level of significance and we conclude that population variances are equal.

Q.10: The following data give the percentage extension under a given load of two independent random samples of yarn, the first before washing, the second after six washing.

Unwashed yarn: 12.3, 13.7, 10.4, 11.4, 14.9, 12.6

Washed yarn 15.7, 10.3, 12.6, 14.5, 12.6, 13.8, 11.9,

Assuming that both samples come from normal distribution test whether here is a significance difference between the two samples:

(a) as regards variability

(b) As regards the mean percentage extension.

Solution:

i) We state our null and alternative hypothesis

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 and σ_2^2 are unbiased estimators of s_1^2 and s_2^2 when n_1 and $n_2 < 120$.

iii) Level of significance

$\alpha = 5\% = 0.05$

iv) Test-Statistic

$F_c = \frac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$$F_c \geq F_{\frac{\alpha}{2}}(v_1, v_2) \geq F_{0.025}(6,5) \geq 6.98 \; and \; F_c \geq \frac{1}{F_{\frac{\alpha}{2}}(v_2, v_1)} \leq \frac{1}{F_{0.025}(5,6)} \leq \frac{1}{5.99} \leq 0.167$$

vi) Calculation

X_1	$\left(X_1-\bar{X}_1\right)^2$	X_2	$\left(X_2-\bar{X}_2\right)^2$
12.3 13.7 10.4 11.4 14.9 12.6		15.7 10.3 12.6 14.5 12.6 13.8 11.9	

$s_1^2=\dfrac{\sum(x_1-\bar{x}_1)^2}{n_1-1}=3.16$ And $s_1^2=\dfrac{\sum(x_2-\bar{x}_2)^2}{n_2-1}=2.57$

$$F_c=\frac{s_1^2}{s_2^2}=\frac{3.16}{2.57}=1.23$$

vii) Conclusion:

Since our calculated value does not fall's in critical region, So we accept H_0 at 5% level of significance and we conclude that there is no difference between the variability of two sample.

b) Solution:

As we see in part one there is no difference between the variability of two samples so we use "t-test" to test whether there significance difference between two sample means percentage.

i) We set up our null and alternative hypothesis

$H_0:\mu_1=\mu_2$

$H_1:\mu_1\neq\mu_2$

ii) Assumption: The two samples of sizes n_1 *and* n_2 are randomly and independently drawn from two normal populations with population mean μ_1 *and* μ_2 when population variances are σ_1^2 *and* σ_2^2.unknown but $\sigma_1^2=\sigma_2^2$ and sample sizes are small.

iii) Level of significance

$\alpha=5\%=0.05$

iv) Test-statistic

$$t_c=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{s_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

If H_0 is true; it hast-distribution with $(V=n_1+n_2-2)$ degree of freedom

v) Critical region

$|t_c|\geq t_{\frac{\alpha}{2}}(v)\geq t_{0.025}(11)\geq 2.201$ $\qquad v=7+6-2=11$

vi) Calculation

$\bar{X}_1=13.06$ And $\bar{X}_2=12.55$

$$S_p=\sqrt{\frac{\sum\left(X_1-\bar{X}_1\right)^2+\sum\left(X_1-\bar{X}_1\right)^2}{n_1+n_2-2}}=1.70$$

$$t_c=\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{s_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}=\frac{(13.06-12.55)-(0)}{1.70\sqrt{\frac{1}{7}+\frac{1}{6}}}=0.54$$

vii) Conclusion

Since our calculated value does not fall's in critical region so we accept $H_0:\mu_A=\mu_B$. And we conclude that the two samples insignificant regard the mean percentage extension at 5% level of significance.

Q.11: The Percent moisture content in a puffed cereal where samples are taken from two different "guns" showed
Gun 1: 3.6, 3.8, 3.6, 3.3, 3.7, 3.4,
Gun 2: 3.7, 3.9, 4.2, 4.2, 4.9, 3.6, 3.5, 4.0
Test the hypothesis of equal variances and equal means, Use any assumptions you believe appropriate.
Solution:
i) We state our null and alternative hypothesis

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} \neq 1 \qquad \text{Or} \qquad H_0 : \sigma_1^2 \neq \sigma_2^2$$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance $\sigma_1^2\ and\ \sigma_2^2$ are unbiased estimators of $s_1^2\ and\ s_2^2$ when $n_1\ and\ n_2 < 120$.
iii) Level of significance
$\alpha = 0.10$
iv) Test-Statistic

$$F_c = \frac{s_2^2}{s_1^2} \qquad \text{Therefore } s_2^2 > s_1^2$$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom
v) Critical region

$$F_c \geq F_{\frac{\alpha}{2}}(v_2, v_1) \qquad \text{if } s_2^2 > s_1^2 \text{ and we use } F_c = \frac{s_2^2}{s_1^2}$$

$$F_c \geq F_{0.05}(7,5) \geq 4.885$$

vi) Calculation

X_1	$(X_1 - \bar{X}_1)^2$	X_2	$(X_2 - \bar{X}_2)^2$
3.6 3.8 3.6 3.3 3.7 3.4		3.7 3.9 4.2 4.2 4.9 3.6 3.5 4.0	

$$s_1^2 = \frac{\sum(x_1 - \bar{x}_1)^2}{n_1 - 1} = 0.034 \qquad \text{And} \qquad s_1^2 = \frac{\sum(x_2 - \bar{x}_2)^2}{n_2 - 1} = 0.20$$

$$F_c = \frac{s_2^2}{s_1^2} = \frac{0.20}{0.034} = 5.88$$

vii) Conclusion:
Since our calculated value fall in critical region. So, we reject H_0 at 5% level of significance and we conclude that variances are not equal.
Therefore variance is not equal, so we do not use t-test to test the equality of means. The assumption of equal variance is necessary to test $H_0 : \mu_1 = \mu_2$

Q.12: Two methods of determining moisture content of samples of canned corn have been proposed and both have been used to make determinations on proportions taken from each of 21 cans. Method 1 is easier to apply but appears to be more variable than method II. If the variability of method 1 were not more then 25 per cent greater than that of method II, we would prefer method 1. Based on the following sample result, which method would you recommend?

$$n_1 = n_2 = 21;\ \bar{y}_1 = 50;\ \bar{y}_2 = 53;\ \sum(y_1 - \bar{y}_1)^2 = 720 \text{ and } \sum(y_2 - \bar{y}_2)^2 = 340$$

Solution:

i) We state our null and alternative hypothesis

$H_0 : \sigma_1^2 \leq 1.25\sigma_2^2$

$H_0 : \sigma_1^2 > 1.25\sigma_2^2$

iii) Assumption: The two samples are drawn independently and randomly from two normal populations with variance σ_1^2 *and* σ_2^2 are unbiased estimators of s_1^2 *and* s_2^2 when n_1 *and* $n_2 < 120$.

iii) Level of significance

$\alpha = 5\% = 0.05$

iv) Test-Statistic

$F_c = \dfrac{s_1^2}{s_2^2}$ Therefore $s_1^2 > s_2^2$

Under H_0; which has F-distribution with $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$ degree of freedom

v) Critical region

$F_c > F_\alpha(v_1, v_2)$

$F_c > F_{0.05}(20,20)$

$F_c > 2.32$

vi) Calculation

$n_1 = n_2 = 21$; $\bar{y}_1 = 50$; $\bar{y}_2 = 53$; $\sum(y_1 - \bar{y}_1)^2 = 720$ and $\sum(y_2 - \bar{y}_2)^2 = 340$

$s_1^2 = \dfrac{\sum(y_1 - \bar{y}_1)^2}{n_1 - 1} = \dfrac{720}{20} = 36$ And $s_2^2 = \dfrac{\sum(y_2 - \bar{y}_2)^2}{n_2 - 1} = \dfrac{340}{20} = 17$

$F_c = \dfrac{s_1^2}{s_2^2} = \dfrac{36}{17} = 1.69$

vii) Conclusion:

Since our calculated value does not fall's in critical region, so, we accept H_0 and we conclude that $H_0 : \sigma_1^2 \leq 1.25\sigma_2^2$ at 5% level of significance.