Estimation

Statistical inference
The process of drawing inferences about population on the basis of sample information that contain the population is called statistical inference.
It has two types
i) Estimation of parameter ii) Testing of hypothesis
Estimation of parameter
Estimation is a procedure to estimate unknown value of population parameter on the basis of sample information.
It has two types
i) Point estimation ii) Interval estimation
Point estimation
The procedure to obtain a single unknown value of population parameter on the basis of sample information by using an estimator is called point estimation.
Point estimate/estimate
A single numerical value calculated from the sample data by using an estimator is called an estimate or point estimate. For example if we have 5, 10,15,20,25.
Then $\overline{X} = \frac{\sum X}{n} = \frac{75}{5} = 15$. Here 15 is a point estimate or estimate and $\frac{\sum X}{n}$ is estimator.
Interval estimation
The procedure to obtain an interval, on the belief that it will include the parameter θ with a known probability is called interval estimation.
Interval estimate
An interval, calculated on the belief that it will include the parameter θ with a known probability is called interval estimate. The confidence interval for θ with probability $(1-\alpha)$ is given by $P(L \le \theta \le U) = (1-\alpha)$ $\quad For\ 0 \le \alpha \le 1$
Where "L" represents lower limit and "U" represent upper limit.
Define estimator
A formula/rule used to estimate the unknown value of population parameter on the basis of sample information is called estimate or estimator. Foe example $\overline{X} = \frac{\sum X}{n}$ here $\frac{\sum X}{n}$ is estimator.
What is meant by interval, confidence interval and confidence coefficient?
Interval: The range of values is called interval
Confidence interval: $(1-\alpha)$ or $100(1-\alpha)\%$ probability is associated with an interval that will contain the population parameter.
Confidence coefficient: An interval to which $100(1-\alpha)\%$ probability is associated that will contain the population parameter
How the width of confidence interval can be decreased?
Ans: The width of confidence interval can be decreased
i) By increasing the sample size
ii) Decreasing confidence coefficient (level of confidence)
If sample size is increased, what will be the change in confidence interval?
Ans: If sample size is increased, the width of confidence interval is decreased.
What will be the effect on a confidence interval if the level of confidence decreases?
Ans: The width of confidence interval decreases if the level of confidence decreases.
Differentiate between estimation, estimate and estimator?
Ans: In statistical inferences, **estimation** is a process; **estimator** is the formula and **estimates** the numerical value.
Differentiate between statistic, estimate and estimator?
Ans: **Estimator** is the formula and **estimates** the numerical value and statistic is a function of random variable being measured.
Why we construct confidence interval?
Ans: Since the point estimates may or may not be representative of the corresponding parameter, its reliability is doubtful. In order to increase reliability, we prefer interval estimation. Hence interval estimation is much more reliable then point estimation. The point estimate is used to obtain an interval estimate.

Define confidence limits.
Ans: The two endpoints of a confidence interval are called confidence limits.
Such as $P(L \le \theta \le U) = (1-\alpha) \qquad For\ 0 \le \alpha \le 1$
Where "L" represents lower limit and "U" represent upper limit.
What is error of estimation?
The distance between estimate and the estimated parameter is called error of estimation.
Define degree of freedom
Degree of freedom is the number of values that are free to vary after we have placed certain restrictions upon the data.
Properties of good estimator
The properties of good point estimator are given below
i) Unbiasedness ii) Consistency iii) Sufficiency iv) Efficiency
v) UMVUE (Uniform minimum variance unbiased estimator) vi) Completeness
vii) Invariance
If these properties satisfy then we say it is good point estimator
Define unbiasedness
The property of an estimator being free from bias is called unbiasedness.
Differentiate between biased estimate and unbiased estimate?
Ans: An estimator is said to be unbiased if the mean of sampling distribution of the statistic is equal to its parameter. $E(\overline{X}) = \mu$ Otherwise it is biased $E(\overline{X}) \ne \mu$
Q.3: Discuss the properties of good point estimator.
Ans: If these properties satisfy then we say it is good point estimator
i) Unbiased
An estimator is said to be unbiased if the mean of sampling distribution of the statistic is equal to its parameter.
$E(\overline{X}) = \mu$
Or
Unbiased estimator
An estimator $\hat{\theta}$ is said to be unbiased estimator of θ if and only if $E(\hat{\theta}) = \theta$ otherwise biased estimator $E(\hat{\theta}) \ne \theta$

i) If $E(\hat{\theta}) = \theta$ it is unbiased estimator
ii) $E(\hat{\theta}) \ne \theta$ it is biased
iii) $E(\hat{\theta}) > \theta$ it is positively biased
iv) $E(\hat{\theta}) < \theta$ it is negatively biased

This property is known as unbiasedness
ii) Sufficiency
An estimator is said to be sufficient estimator if it utilize all the values which are contained in the sample and the property is possess known as sufficiency.
iii) Consistency
An estimator $\hat{\theta}$ is said to be consistent estimator if it tends to parameter θ as $n \to \infty$
i) $\lim_{n\to\infty} P\left[\left|\hat{\theta}-\theta\right| \le \in\right] = 1$ **ii)** $\lim_{n\to\infty} Var(\hat{\theta}) = 0$
iv) Efficiency
If $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbiased estimator then if $Var(\hat{\theta}_1) < Var(\hat{\theta}_2)$ then $\hat{\theta}_1$ is called the efficient estimator as compared to $\hat{\theta}$
Q.4: Precision and accuracy.
Ans: Precision is the size of the deviation from the repeated sample mean whereas accuracy is the size of the deviation from the overall population mean.
Q.2: Define mean square error and prove that $M.S.E(\hat{\theta}) = Var(\hat{\theta}) + (Bias)^2$
Ans: Mean square error (MSE)
In sample survey we have to get different estimate of population parameter and we want to see which estimate is very closer to the true value of population parameter. We use criteria of a mean square error, which is different from the expectation of the square differences of an estimate and the true value.
$M.S.E(\hat{\theta}) = E(\hat{\theta}-\theta)^2$

Proof:
Let by definition

$M.S.E(\hat{\theta}) = E(\hat{\theta} - \theta)^2$

Adding and subtracting $E(\hat{\theta})$, we get

$M.S.E(\hat{\theta}) = E\left[\hat{\theta} - E(\hat{\theta}) + E(\hat{\theta}) - \theta\right]^2$

$M.S.E(\hat{\theta}) = E\left[(\hat{\theta} - E(\hat{\theta})) + (E(\hat{\theta}) - \theta)\right]^2$

$M.S.E(\hat{\theta}) = E\left[\left(\hat{\theta} - E(\hat{\theta})\right)^2 + \left(E(\hat{\theta}) - \theta\right)^2 + 2\left(\hat{\theta} - E(\hat{\theta})\right)\left(E(\hat{\theta}) - \theta\right)\right]$

$M.S.E(\hat{\theta}) = E\left(\hat{\theta} - E(\hat{\theta})\right)^2 + E\left(E(\hat{\theta}) - \theta\right)^2 + 2E\left(\hat{\theta} - E(\hat{\theta})\right)\left(E(\hat{\theta}) - \theta\right)$

$M.S.E(\hat{\theta}) = Var(\hat{\theta}) + \left(Bias(\hat{\theta})\right)^2 + 2(0)$

$M.S.E(\hat{\theta}) = Var(\hat{\theta}) + \left(Bias(\hat{\theta})\right)^2$ Therefore $E\left\{\hat{\theta} - E(\hat{\theta})\right\} = \left\{E(\hat{\theta}) - E(\hat{\theta})\right\} = 0$

If E is an unbiased estimator then $E(\hat{\theta}) - \theta = 0$

Then

$M.S.E(\hat{\theta}) = Var(\hat{\theta})$

Explain the process of efficiency of comparison for unbiased as well as biased estimators.

Solution:

For Unbiased estimator

If $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbiased estimator of parameter θ_1 and θ_2. Then $\hat{\theta}_1$ is called more efficient estimator of θ_1 if $\mathrm{Var}(\hat{\theta}_1) < Var(\hat{\theta}_2)$. Then relative efficiency of unbiased estimator is given as

$$\mathrm{Re}lative\ efficiency = \frac{Var(\hat{\theta}_1)}{Var(\hat{\theta}_2)} \times 100$$

For biased estimator

In this situation we find the mean square error for these estimator as

$M.S.E(\hat{\theta}_1) = Var(\hat{\theta}_1) + \left(Bias(\hat{\theta}_1)\right)^2$

$M.S.E(\hat{\theta}_2) = Var(\hat{\theta}_2) + \left(Bias(\hat{\theta}_2)\right)^2$. Then $\hat{\theta}_1$ is called more efficient estimator of θ_1 if $\mathrm{M.S.E}(\hat{\theta}_1) < M.S.E(\hat{\theta}_2)$. Then relative efficiency of biased estimator is given as

$$\mathrm{Re}lative\ efficiency = \frac{\mathrm{M.S.E}(\hat{\theta}_1)}{\mathrm{M.S.E}(\hat{\theta}_2)} \times 100$$

Define the likelihoods function

The likelihood of "n" random variable is defined to be the joint density of "n" variables. Let "$X_1, X_2, X_3, \ldots, X_n$" be a random sample of size "n" from a density $f(X, \theta)$. Then L.H.F is defined as

$$\mathrm{L}(\mathrm{X}, \theta) = \mathrm{f}(\mathrm{X}_1, \theta), \mathrm{f}(\mathrm{X}_2, \theta), \mathrm{f}(\mathrm{X}_3, \theta), \ldots, \mathrm{f}(\mathrm{X}_n, \theta) = \underset{i=1}{\overset{n}{\pi}} \mathrm{f}(\mathrm{X}_i, \theta) = \mathrm{L}(\underline{X}, \theta) = \mathrm{L}(\underline{X})$$

Describe the mathematical technique of finding maximum likelihood function

First of all take the likelihood function of the p.d.f then take the log likelihood function. Then differentiate log likelihood function with respect to the parameter and put equal to zero. Then obtain the estimate of the respective parameter and it is called maximum likelihood estimator.

$$\frac{\partial \log \mathrm{L}(\mathrm{X}, \theta)}{\partial \theta} = 0$$

$Then\ \ obtain\ \hat{\theta}$